Energy balance in a simplified rowing model

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1 Introduction

The model of boat, rower and oars as presented before (Simulation, model and results), to my opinion, quite satisfactorily solves the equation of motion and gives a realistic shell velocity graph.
The energy balance is not quite clear however. Initially I calculated the outflow of energy along the shell and at the blade. My opinion was that the sum of the two was the total energy involved and consequently this was the energy to be delivered by the rower. Later Atkinson convinced me that the rower moving on his seat generated and dissipated additional energy. See Kinetic energy dissipated by the moving rower. The question is: how much of the kinetic energy accumulated in the rower's mass is dissipated in the mechanism of the rower and how much is eventually used for the propulsion of the boat. In order to get a better insight into this question a simpler model has been constructed and reported below with some results.
Anticipating on results to be described that dissipation of kinetic energy in this simple model occurs but in a racing shell with normal mass distribution it is negligible.
In my model of boat and rower the force in neither the rower's legs nor the acceleration of the mass m_2 is available but it can be extracted from the simulation results and an energy balance has been obtained. (Go home and select "Energy balance of the model as described in item 1").

The results obtained in the present model do not immediately support the conclusions in Kinetic energy dissipated by the moving rower. A weak point is this latter web page is that in considering the recovery without hull friction, the hull velocity at the start and at the end of the recovery is the same, which is a coarse assumption.

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2 Model

The model consists of two masses:

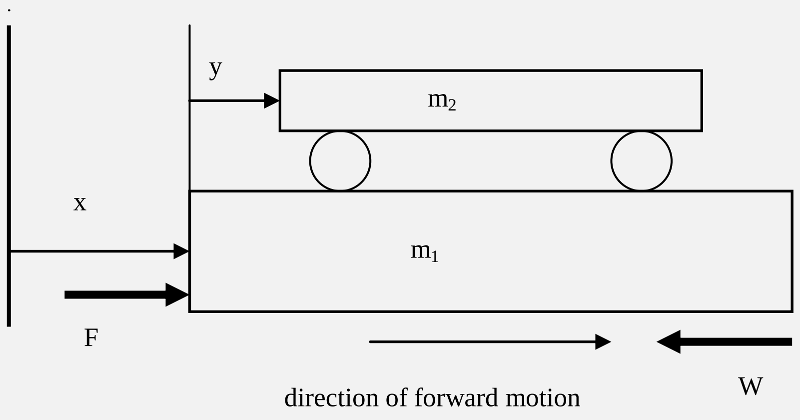

Fig 2.1
Model of boat and rower

m_1= mass of the boat + that part of the mass of the rower that does not move with respect to the boat + hydrodynamic added mass
m_2= that part of the mass of the rower that moves with respect to the boat
x = absolute coordinate of the boat
y = relative coordinate of the mass m_2 with respect to the mass m_1
z = absolute coordinate of the joint centre of mass of m_1 and m_2

The propulsion is very simple. A force F is acting on the shell if $\dot{y} > 0$ or $F = F*(\dot{y} > 0.)$. That is when m_2 is moving towards the bow.
A resistance force acts on the shell $W = C.\dot{x}^2$.

The relation between x, y and z is given by:

$$z(m_1 + m_2) = xm_1 + (x + y)m_2$$

$$z = \frac{m_1}{m_1 + m_2}x + \frac{m_2}{m_1 + m_2}(x + y)$$

$$z = x + \frac{m_2}{m_1 + m_2}y$$

and:

$$x = z - \frac{m_2}{m_1 + m_2}y$$

The motion of the mass m_2 with respect to the mass m_1 is prescribed by $\dot{y} = \frac{\pi L}{T}\sin\frac{2\pi}{T}t$ and entered in the model as

$$\ddot{y} = 2L\left(\frac{\pi}{T}\right)^2 \cos\frac{2\pi}{T}t$$

L = track length (sliding length)
T = cycle time
t = time

This results in:
y = 0 for t = 0 + k.T
y= L for t = 0.5T + k.T
(k is an arbitrary whole number)

The basic equation is

$$\ddot{z} = \frac{1}{m_1 + m_2}(F - W) \quad \text{(Newton's second law)}$$

From

$$x = z - \frac{m_2}{m_1 + m_2} y$$

follows

$$\ddot{x} = \ddot{z} - \frac{m_2}{m_1 + m_2}\ddot{y} = \ddot{z} - \frac{m_2}{m_1 + m_2} 2L\left(\frac{\pi}{T}\right)^2 \cos\frac{2\pi}{T}t$$

The solution has been obtained by a Scilab program using an explicit integration method.

$$A1 = \frac{1}{m_1 + m_2}$$

$$A2 = \frac{m_2}{m_1 + m_2}$$

$$C1 = 2L\left(\frac{2\pi}{T}\right)^2$$

The definition of these constants is of interest only for those reading the program code.

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3 Additional calculations

3.1 Power

To develop an energy balance we first develop a power (energy flow) balance and distinguish the generation of power, the outflow of power and the increase/decrease of kinetic energy. Besides the external force F, the force Q that moves mass m_2 with respect to mass m_1 is a source of energy.
See Fig 3.1 below.

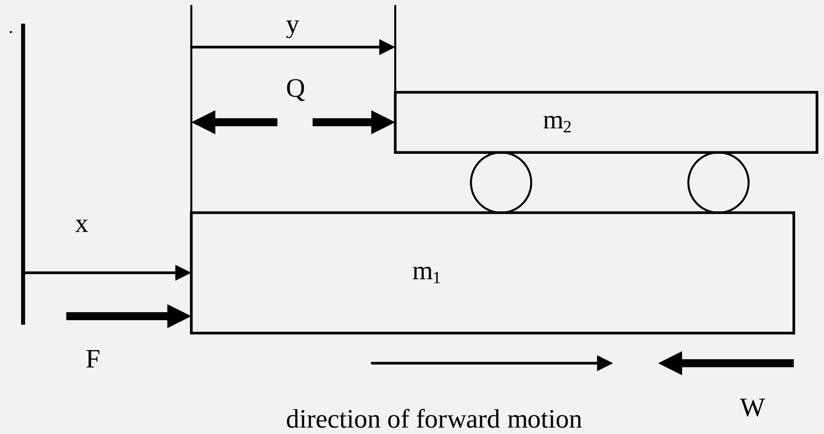

Fig 3.1
The force Q

The force Q follows from:

$$Q = m_2(\ddot{x} + \ddot{y})$$

The power delivered by Q is:

$$PQ = Q.\dot{y} = m_2(\ddot{x} + \ddot{y})\dot{y}$$

The power delivered by the external propulsion force F is

$$PF = F.\dot{x}$$

The power dissipated at the hull, $PW = W.\dot{x}$.

The kinetic energy in the system is E = Em_1 + Em_2 where
Em1 = kinetic energy in mass m_1
Em2 = kinetic energy in mass m_2

For every instant of time must hold:

$$PF - PW = \frac{dEm_1}{dt} + \frac{dEm_2}{dt}$$

with

$$Em_1 = 0.5\ m_1.\dot{x}^2$$

$$Em_2 = 0.5\ m_2(\dot{x} + \dot{y})^2$$

and

$$Pm_1 = \frac{dEm_1}{dt} = m_1.\dot{x}.\ddot{x}$$

$$Pm_2 = \frac{dEm_2}{dt} = m_2.(\dot{x} + \dot{y}).(\ddot{x} + \ddot{y})$$

Power balance at every instant of time within one cycle

$PF + PQ = PW + Pm_1 + Pm_2$

3.2 Energy

The energy delivered/dissipated during one cycle is considered. The zero level of the kinetic energy is the kinetic energy present at the start of the cycle and this equals, for a stationary situation, the kinetic energy at the end of the cycle. So, a negative reading for the kinetic energy in Fig 4.5 is possible.

The energy delivered by the external propulsion force is

$$EF(t) = \int_0^t PF(t).dt$$

The energy delivered by the internal force Q is

$$EQ(t) = \int_0^t PQ(t).dt$$

The energy dissipated during the cycle is

$$EW(t) = \int_0^t PW(t).dt$$

The kinetic energy in the system during the cycle is

$$Em_1(t) = \int_0^t Pm_1(t).dt$$

$$Em_2(t) = \int_0^t Pm_2(t).dt$$

For the total energy delivered/dissipated during one cycle, take t = T

These calculations are carried out in Scilab.

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4 Results of simulation

4.1 Results for one set of input data

Input data

L = 0.8 m
T = 2.0 s
m1 = 40.0 kg
m2 = 60.0 kg
C2 = 3.0 $N.s^2.m^{-2}$
F = 160.0 N

Results of the simulation are:

F	vel	PWmean	PFmean	PQmean
[N]	[m/s]	[W]	[W]	[W]
160	5.12	418	373	46

Table 4.1

where vel is the mean hull velocity during the cycle.

The energies are expressed as mean power during one cycle; it is the exchanged energy during one cycle divided by the cycle time. See also the remark above Fig 4.3.

PFmean and PQmean must sum up to Pwmean. Here we miss 1W. This is considered as loss of accuracy in the integration. PQmean is the result of integration of PQ, the blue line in Fig 4.2 taking into account the negative values PQ (the area below the zero level in Fig 4.2). In this chapter we will accept this as the correct way of dealing with PQ.

Fig 4.1 gives the velocity of the hull during one cycle. Note that the start velocity equals the end velocity.

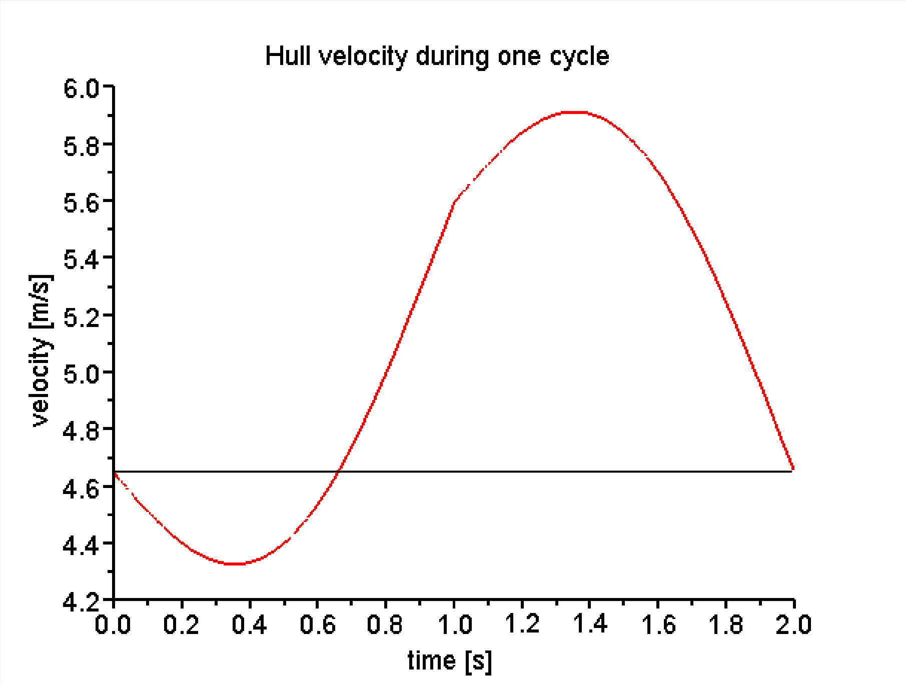


Fig 4.1
Hull velocity variation

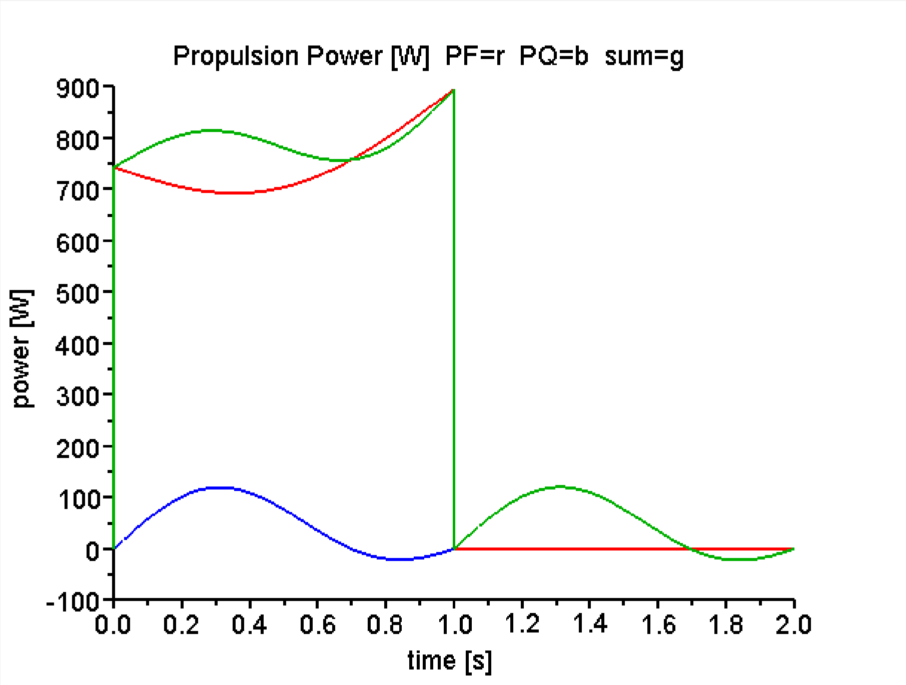


Fig 4.2
The power of the propulsion force and the resistance during one cycle.

red line = PF, zero during recovery
blue line = PQ, covered by the green line during recovery
green line = sum of PF and PQ

The power delivered by the propulsion force during the first half of the cycle reaches a maximum value of 900 W. See Fig 4.5. Remember that this is an instantaneous value. In rowing (and readings on the ergo display) usually mean values over one cycle are considered. That value is presented in Table 4.1 as PFmean and the same for PQ.

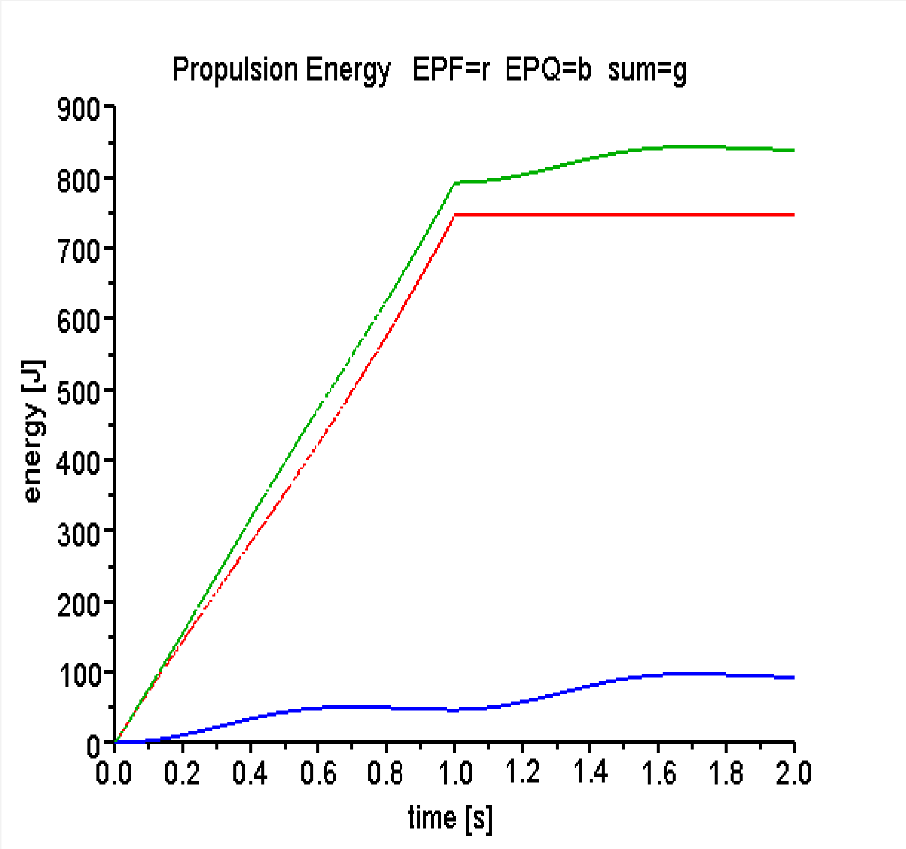


Fig 4.3
Energy delivered by the propulsion force and dissipated energy due to hull resistance.

red line = EPF
blue line = EQF
green line = EPF + EQF

Fig 4.3 makes clear that the outflow of energy is greater than the energy delivered by the propulsion force. There must be another energy source. The only candidate is the Q force that moves m_2 with respect to m_1. This energy indeed balances in- and outflow of energy as already mentioned in the beginning of this chapter, the blue line.

Now the outflow of energy and change of kinetic energy is considered. See Fig 4.4

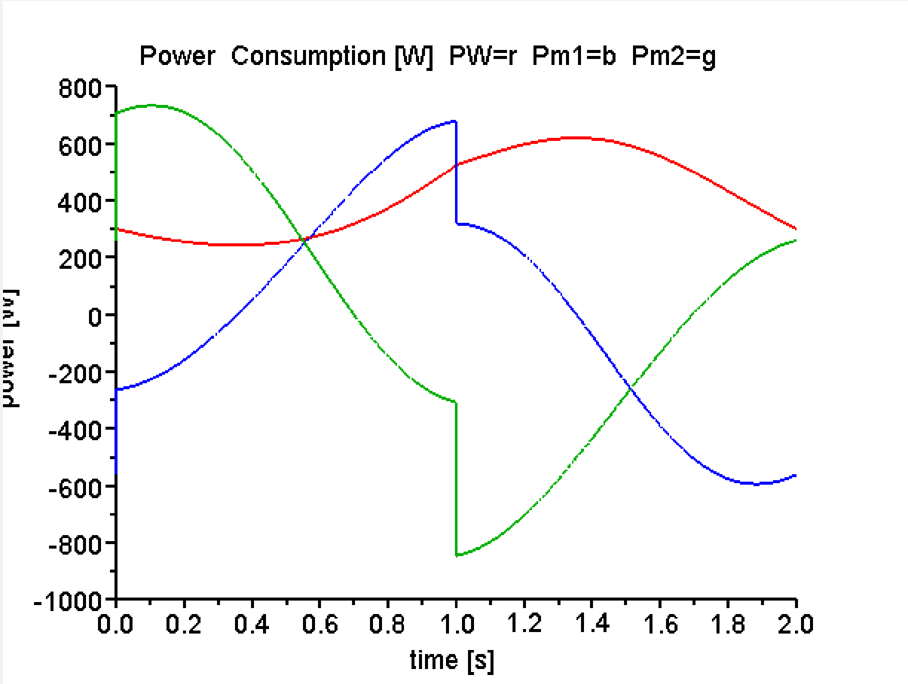


Fig 4.4
Power outflow and change of kinetic energy

red line = PW outflow of power
blue line = Pm_1 change of kinetic energy in mass m_1
green line = Pm_2 change of kinetic energy in mass m_2

In Fig 4.5The outflow of energy and the quantity of kinetic energy is presented. The graph shows negative quantities of kinetic energy during certain phases of the cycle. This is possible because the zero level of kinetic energy has been taken as the kinetic energy at the start of the cycle, as observed before.

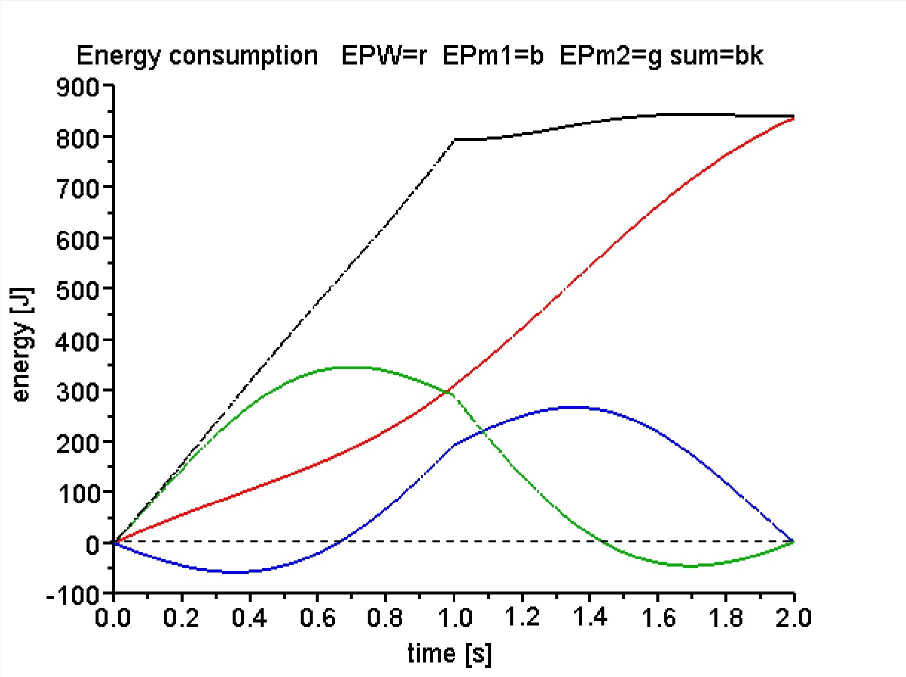


Fig 4.5
Energy outflow and amount of kinetic energy

red line = EPW outflow of energy
blue line = EPm_1 amount of kinetic energy in mass m_1
green line = EPm_2 amount of kinetic energy in mass m_2
black line = $EPW + Epm_1 + Epm_2$

Note that the black line in Fig 4.5 is the same as the green line in Fig 4.3.

At this point it is concluded that an energy balance has been obtained that it is in equilibrium. Inflow and outflow have the same magnitude.

In Fig 4.2 the blue line shows the power supply to the system by the force Q. The power supply by Q is shown again in a separate chart, see Fig 4.6, a and b. This power is most of the time positive. It contributes to the propulsion of the system. Only during short periods it has a negative value. Because the mechanism that moves m_2 with respect to m_1 as modelled here is a conservative power source. Positive power is delivered (= generated), negative power flows back to the mechanism (=absorbed). A rower however is not a conservative mechanism. He cannot absorb energy. He can only dissipate it. Compare a human being going downstairs. Considering the total energy delivered by the rower we should not subtract the energy under the zero line. On the contrary, as going downstairs also requires effort (although less than going upstairs) we should add some quantity to the energy delivered by the rower. We only don’t know how much. Now we make the arbitrary choice to add the same amount of energy as is subtracted in the conservative system.
It is clear that in that case the equilibrium in the energy balance will be disturbed. It can be brought back to equilibrium to assign the difference to internal dissipation.
The negative value of EPQ, expressed as mean power, is rather small in this model (it is only 9W) in contrast to what is found by Atkinson. He has found a considerable

amount of energy dissipated by the motion of the rower. The results of the simplified model cannot directly be applied to a detailed model of boat, oars and rower.

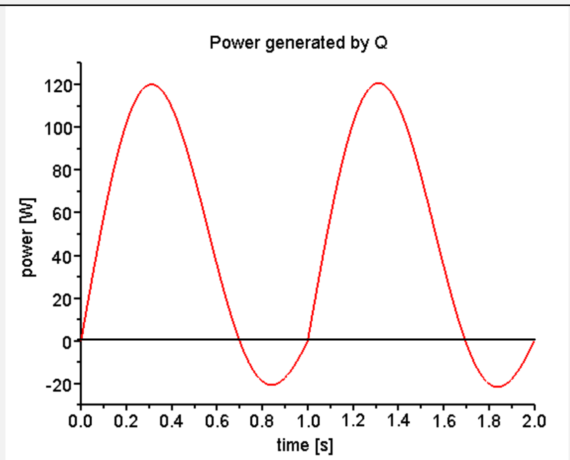
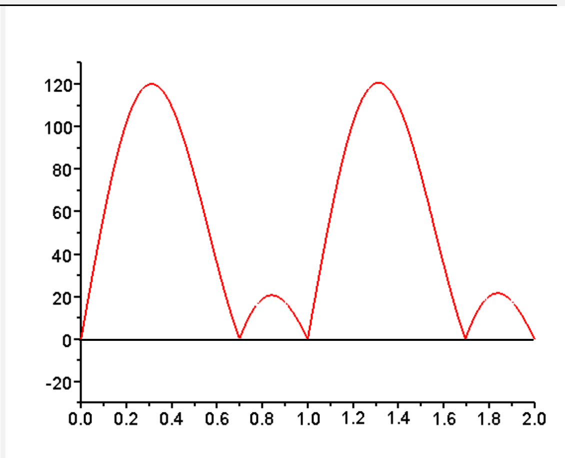


Fig 4.6 a
Power generated by force Q
Negative power recovered



Negative Fig 4.6 b
Power generated by force Q
power reversed

	F	vel	PWmean	PFmean	PQmean	diss.
	[N]	[m/s]	[W]	[W]	[W]	[W]
cons	160	5.12	418	373	46	0
non-cons	160	5.12	418	373	55	9

Table 4.2

In Table 4.2 the results for a conservative system, Fig 4.6 a, are shown in the row marked “cons’’ and are identical to Table 4.1.
The row marked “non-cons” contains the results of the system represented by Fig 4.6 b. The extra dissipated energy is given in the column marked “diss.”.

4.2 Results for different values of the propulsion force

F	vel	PWmean	PFmean	PQmean
[N]	[m/s]	[W]	[W]	[W]
120	4.42	271	237	36
160	5.12	418	373	46
200	5.73	584	529	57

Table 4.3
PWmean = PFmean + PQmean for several values of F.
(negative power by the force Q accepted)

Table 4.3 shows that the PQ power is relatively less at high values of the propulsion force.

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5 Conclusion1

For the presented model the energy generated to shift the mass m_2 (= rower) forward and backwards with respect to mass m_1 (= boat) contributes to the propulsion. The amount of energy is relatively small. The difference with a rowing model is that there the propulsion energy is largely delivered by the legs. During the recovery however, the difference between the two situations does not exist. The conclusion of the this exercise might be that during the recovery

- rower's motion contributes to the propulsion
- only a small quantity of energy is dissipated (the right shaded area in Fig 4.4)

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The next chapters refer to the previous Matlab/Simulink results. Scilab recalculations are still pending.

6 Continuation of the simulation. Another velocity function of m2 relative to m1.

In the previous sections the relative velocity of m2 with respect to m1 (seat velocity) was a sine function, see section 2: $\dot{y} = \frac{\pi L}{T} \sin \frac{2\pi}{T} t$.

With a.o. Atkinson I am of opinion that the seat velocity function, *constant recovery time assumed*, does not influence the system's speed. But one can wonder whether the energy consumption is also invariant for the seat velocity function. This simplified model seems to be a good tool to investigate this question. The basic seat velocity is shown in Fig 6.1.

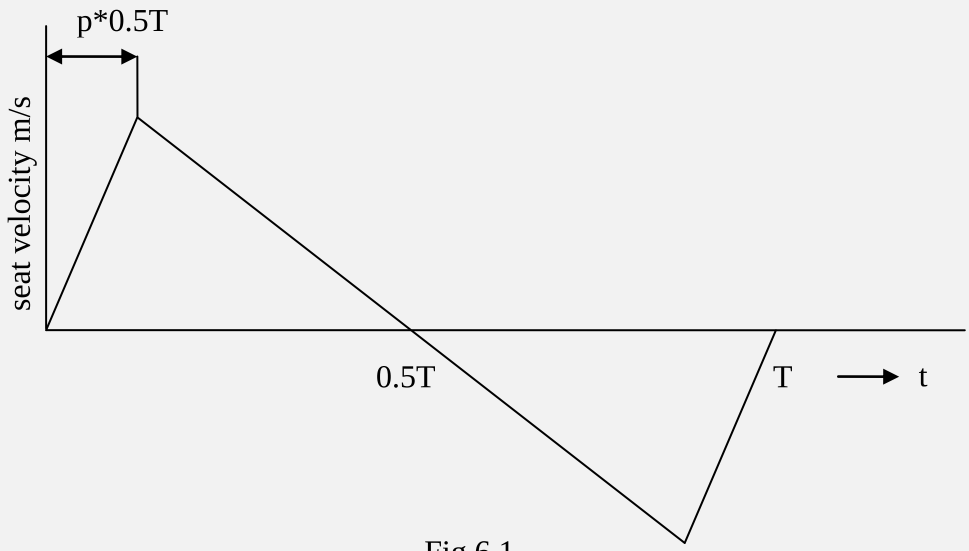

Fig 6.1
Seat velocity function

The maximum velocity follows from:

$$L = 0.25.T.v_{max}$$

$$v_{max} = \frac{4L}{T}$$

The parameter p defines when the maximum seat speed is reached. 0<p<1.

Compare: for the seat speed as a sine function as used above $v_{max} = \frac{\pi L}{T}$

We compare now the triangular seat speed with p = 0.5 and F =160N with the sinusoidal seat speed.

	F	vel	PWmean	PFmean	PQmean	PQRmean
	[N]	[m/s]	[W]	[W]	[W]	[W]
sinus	160.00	5.13	421	374	47	55
triangle	160.00	5.10	415	367	47	64

Table 6.1
Comparison of results of sinusoidal and triangular seat speed

The hull velocity variation during one cycle for the triangular seat speed is shown in Fig 6.2. Compare with Fig 4.1.

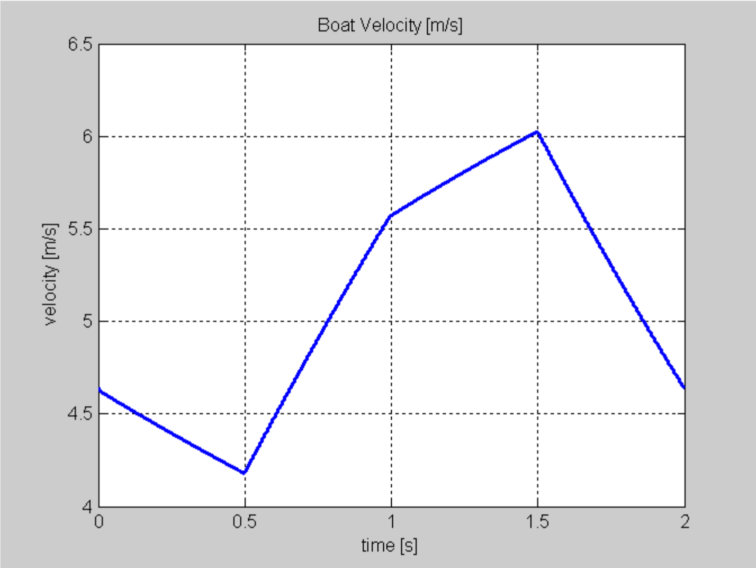

Fig 6.2
Hull velocity variation

Observe that the maximum hull speed is higher and the minimum hull speed is lower than for the sinusoidal seat speed.
Table 6.1 shows few differences in speed and energy with the exception of PQRmean that remains the same.

The other graphs are presented below. In spite of considerable differences in instantaneous hull velocity and power functions, the mean hull velocity and the integrals of the power functions show remarkable conformity with those of the sinusoidal seat speed.

For comparison the power generated by the force Q is presented in Fig 6.5.

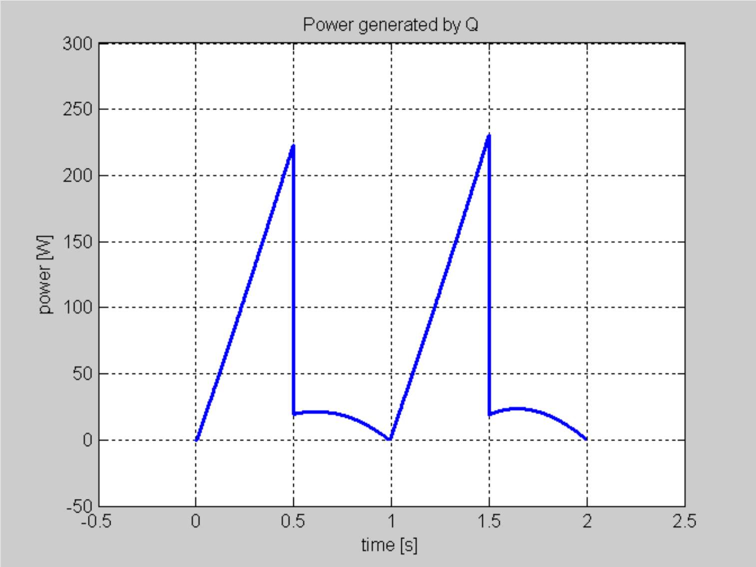

Fig 6.5
Power of the force Q
Negative power reversed. p = 0.5

To conclude this discussion the results for various values of the parameter p are presented in Table 6.2

F	p	vel	Pwmean	Pfmean	PQmean	PQRmean
[N]	[-]	[m/s]	[W]	[W]	[W]	[W]
160	0.1	5.16	428	381	46	73
160	0.2	5.12	418	372	46	70
160	0.3	5.14	423	375	48	66
160	0.4	5.11	418	370	47	64
160	0.5	5.10	415	367	49	64
160	0.6	5.14	423	376	47	64
160	0.7	5.12	419	372	47	66
160	0.8	5.13	423	374	49	70
160	0.9	5.09	413	365	48	73

Table 6.2
Results for various values of the parameter p

It is difficult to discover a trend in the quantities as a function of p. Values have been checked but no error was discovered.

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7 Conclusion2

The exercise described in section 6 confirms that the function of the seat motion with respect to the boat has little influence on the mean boat velocity, provided that the frequency is constant.

It is remarkable that in this simplified model, where the seat motion is in no way connected with the propulsion, it contributes in spite of that to the propulsion.

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