Energy balance of the model as described in “Theory and Model”

1. Introduction

The energy balance as presented below is based on energy (or power) sources and –sinks and avoids such notions as power through the foot stretcher or through the oarlock. My simulation model does not contain internal accelerations and forces in the system and so I am obliged to differentiate boat and rower velocity. Differentiation in numerical simulation is in general not considered as good practice. It results in noise and spikes. Because in the end these signals are integrated again the noise disappears again.
The equation of motion is solved with SimmodN.sce as before.

2. Power sources and –sinks

The rower is the source of power. This source can be split into three sub sources: legs, back and arms. I order to keep it simple I consider legs and arms only. The rower is someone with a stiff back. Not ideal but for the time being acceptable, it is just the idea and the approach I want to make clear.
For me it is helpful to look at legs and arm as hydraulic cylinders that deliver energy by extending (legs) and contracting (arms) during the drive.
The legs deliver energy when extending and the internal force is a compression force and absorb energy when either the extending becomes contracting or (not and) the internal force shifts from compression to tension.
The arms deliver energy when contracting and the internal force is a tension force. They absorb energy when either the contracting becomes extension or (not and) the tension becomes compression. In the simulation delivered energy is positive and absorbed energy is negative. The final amount of work delivered by the rower is the algebraic sum of the positive and negative work, the model is internally conservative. In the physical reality the positive amount of work is delivered by the rower, the negative work is lost. The rower is not a conservative but dissipative system. In a perfect simulation the amount of energy produced by the rower (algebraic sum of leg- and arm work) equals the outflow of energy as hull friction plus the outflow at the blade (puddle energy). For the human rower the negative work calculated in the model has to be added to the work delivered by him. See the final balance sheet at the end, Table 3.1.

Now look at the rower model Fig 2.1 (I hope the reader can distinguish the anatomy of the rower):

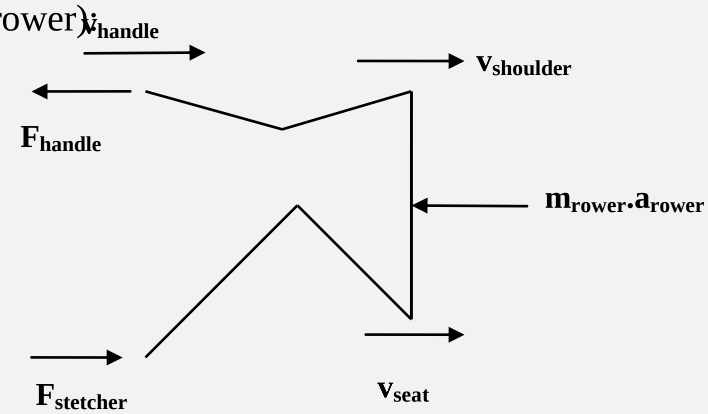

Fig 2.1
Rower model, legs, arms and back.

The forces are the forces ***on*** the rower. The velocities are the velocities with respect to the boat (any other reference system will do because only velocity differences will appear in my reasoning). The acceleration $\mathbf{a}_{\mathbf{rower}}$ of the rower's mass $\mathbf{m}_{\mathbf{rower}}$ is measured in a world fixed reference system.

Power delivered by the legs: $\mathbf{P}_{\mathbf{legs}} = \mathbf{F}_{\mathbf{stretcher}} \cdot \mathbf{v}_{\mathbf{seat}}$

Power delivered by the arms: $\mathbf{P}_{\mathbf{arms}} = \mathbf{F}_{\mathbf{handle}} \cdot (\mathbf{v}_{\mathbf{handle}} - \mathbf{v}_{\mathbf{shoulder}})$

Total power: $\mathbf{P}_{\mathbf{tot}} = \mathbf{P}_{\mathbf{legs}} + \mathbf{P}_{\mathbf{arms}}$

This is all what is to say about power delivered by the rower. There is no need to introduce -stretcher or gate power.

(For further discussion on the power balance, see below)

Relation between $\mathbf{F}_{\mathbf{stretcher}}$ and $\mathbf{F}_{\mathbf{arms}}$: $\mathbf{F}_{\mathbf{arms}} = \mathbf{F}_{\mathbf{stretcher}} - \mathbf{m}_{\mathbf{rower}} . \mathbf{a}_{\mathbf{rower}}$

power delivered = power outflow + increase of kinetic energy (no potential energy is involved in rowing) for the situation that a blade is in the water.

power outflow P_{out} = power dissipated by fluid friction along the boat + power dissipated at the blade (= work done on the water by the blade force)

$P_{out} = \mathbf{C} . \mathbf{v}_{\mathbf{boat}}^{3} + \mathbf{F}_{\mathbf{blade}} \cdot \mathbf{v}_{\mathbf{blade}}$ (this last term to be interpreted as a vector dot product)

$\mathbf{C}$ = a constant, $\mathbf{v}_{\mathbf{boat}}$ = boat velocity with respect to the water and $\mathbf{v}_{\mathbf{blade}}$ is the blade velocity with respect to the water.

The rate of change of kinetic can be written, in general as:

$$\frac{dE_k}{dt} = \frac{d(0.5\, m\, v^2)}{dt} = m.v\,\frac{dv}{dt} = m.v.a$$

$$\frac{dE_k}{dt} = \mathbf{m_{boat} \cdot v_{boat} \cdot a_{boat} + m_{rower} \cdot v_{rower} \cdot a_{rower}}$$

All velocities and accelerations with respect to an earth bound system = with respect to the water. Summarized, the power to be delivered by the rower during the drive is:

$$\mathbf{P_{tot} = C.v_{boat}^3 + F_{blade} \cdot v_{blade} + m_{boat} \cdot v_{boat} \cdot a_{boat} + m_{rower} \cdot v_{rower} \cdot a_{rower}}$$

3. Numerical results

In order to understand the following results is necessary to familiarise with the model. See "Theory and Model" and "Simulation Results". The drag- and lift coefficients used follow the description of Caplan and Gardner. See the web page Lift and Drag.

******* initial data *********

m1	m2	Fbl	L	fi1	fi2	sl	TR
kg	kg	N	m	rad	rad	m	s
30.0	70.0	400	1.80	-1.10	0.60	0.80	1.10

area	C1	C2
m^2	$N.s^2/m^2$	-
0.130	3.5	1.0

------- results -------

Ebls	Exs	Et	eff	T
J	J	J	-	s
203.1	747.9	951.0	0.786	1.86

SR	T2000	Prow	vel
m^{-1}	s	W	m/s
32.26	421.3	511.27	4.748

Table 3.1
Input data and results of simulation

m_1=	mass of the boat + that part of the mass of the rower that does not move with respect to the boat + hydrodynamic added mass
m_2=	that part of the mass of the sculler that moves with respect to the boat
Fbl =	force perpendicular on blade
L=	distance pin to point of application of force on blade
fi1=	value of φ at the catch
fi2=	value of φ at the finish
sl =	distance covered by m_2 with respect to m_1 (sliding length)
TR=	time for the recovery
area=	area of two blades
C1=	resistance coefficient of boat hull
C2=	maximum lift coefficient of blade (actual value depends on angle of attack)
Ebls=	energy delivered at the blade
Exs=	energy dissipated by boat resistance
Et=	sum of Ebls and Exs
eff=	overall efficiency
T=	time for one stroke
SR=	strokes per minut
T2000=	time to cover 2000m
Prow=	total energy flow (power) to be delivered to the system
vel=	mean velocity of the boat

Table 3.2
Explanation of used names

In the former simulation the inboard handle length was not specified. The present results are obtained with an inboard length Li = 0.9 m. Li can be chosen arbitrarily without effecting the simulation results. It only effects the energy distribution and not the bottom line of the energy balance. A longer inboard length results in more arm energy and less leg energy and the reverse.

Fig 3.1 shows the normalised blade force. The maximum is set at 400 N.
Fig 3.2 shows the seat speed during the recovery.

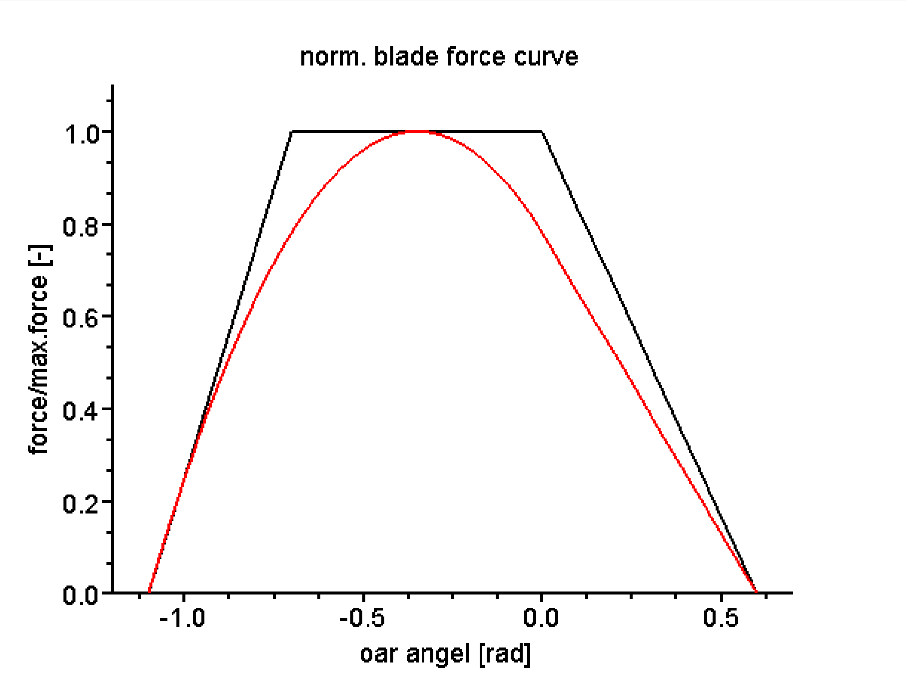

Fig 3.1
Normalised blade force

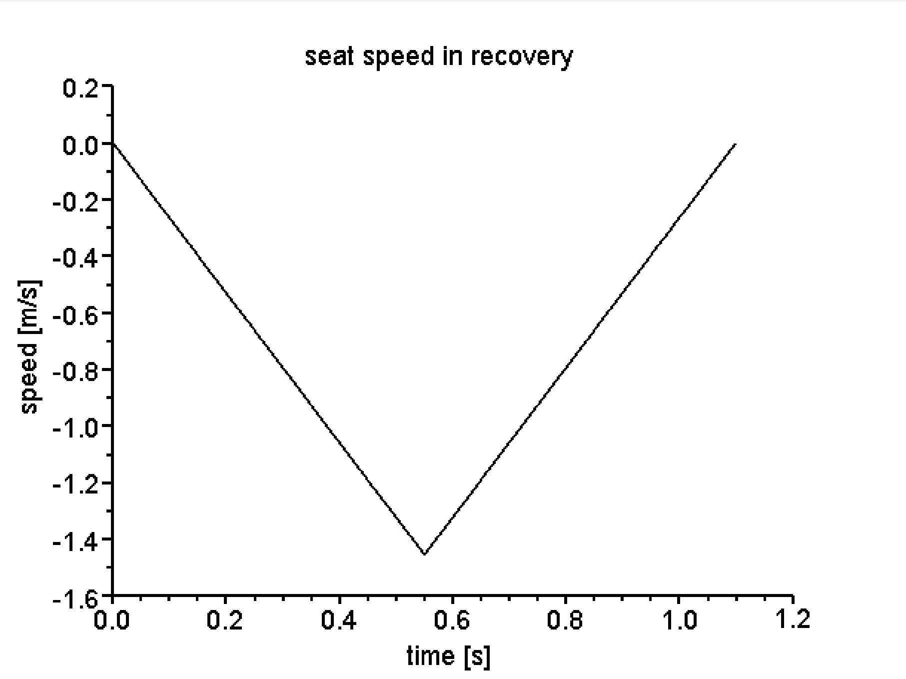

Fig 3.2
Seat speed during recovery

□

The amount of energy Ebls (energy dissipated at the blade) and Exs (energy outflow due to hull friction) has been summed to Et = 951.0 J. This follows immediately from the simulation.

In the present calculation the total energy derived from legs and arms = 951.8 J. The energy calculated as outflow, including the storage as kinetic energy = 953.4 J.
These three numbers should be the same but the resemblance is satisfactory.

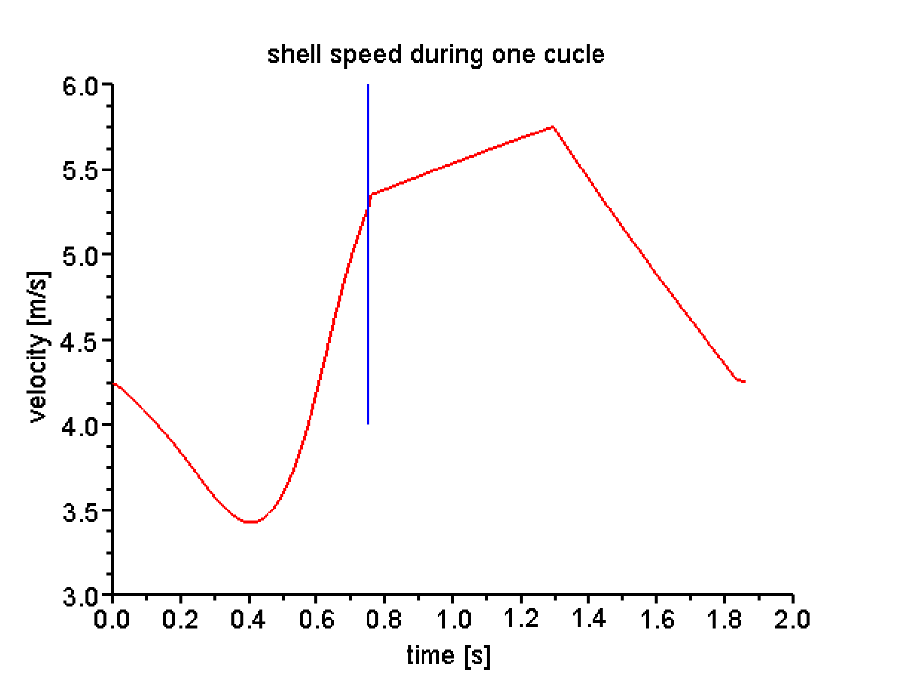

Fig 3.3
Hull speed during one cycle.
Vertical line marks the transition drive-recovery.

In Fig 3.4 the accelerations are presented. Te green line is the acceleration of the common centre of mass and is directly produced by the SimmodN program. The red and the green line are the products of differentiation. The result was fouled by noise and spikes. By smoothing the curves have been cleaned.

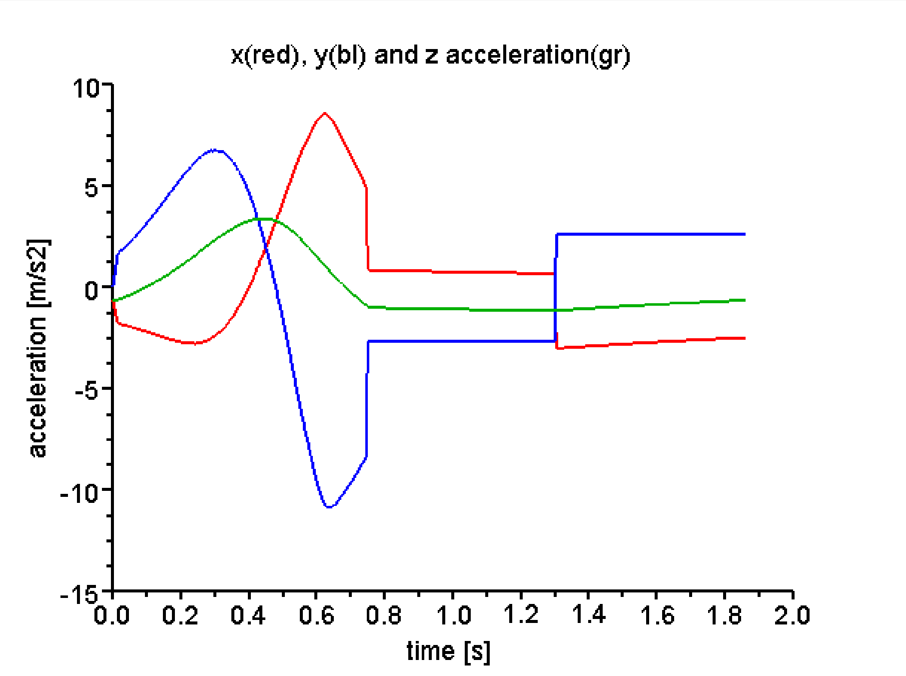

Fig 3.4 Accelerations:
blue: rower
green: system
red: hull

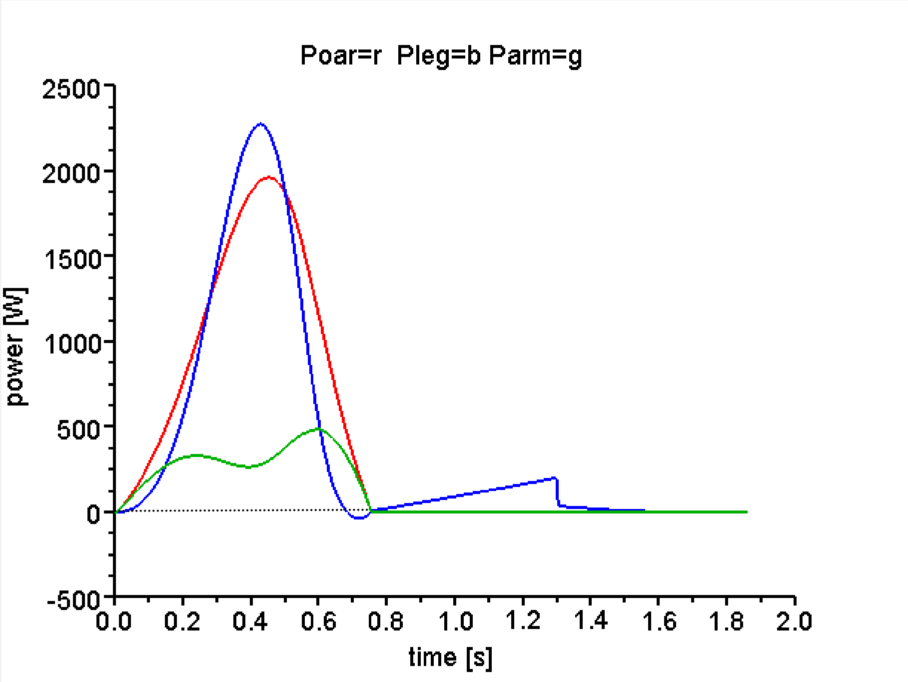

Fig 3.5
Power generated by arms and legs.
green: power generated by the arms
blue line: power generated by the legs
red line: power delivered at the handle

In the following charts the title in the top may look rather cryptic. Consider them indications for the author. Under the charts an understandable title has been written.

Fig 3.5 displays the power generated by arms and legs and the power delivered at the oar that is not the sum of legs and arms power. Note that the legs' power is considerable more than the arms' power.

The difference is shown in Fig 3.7.

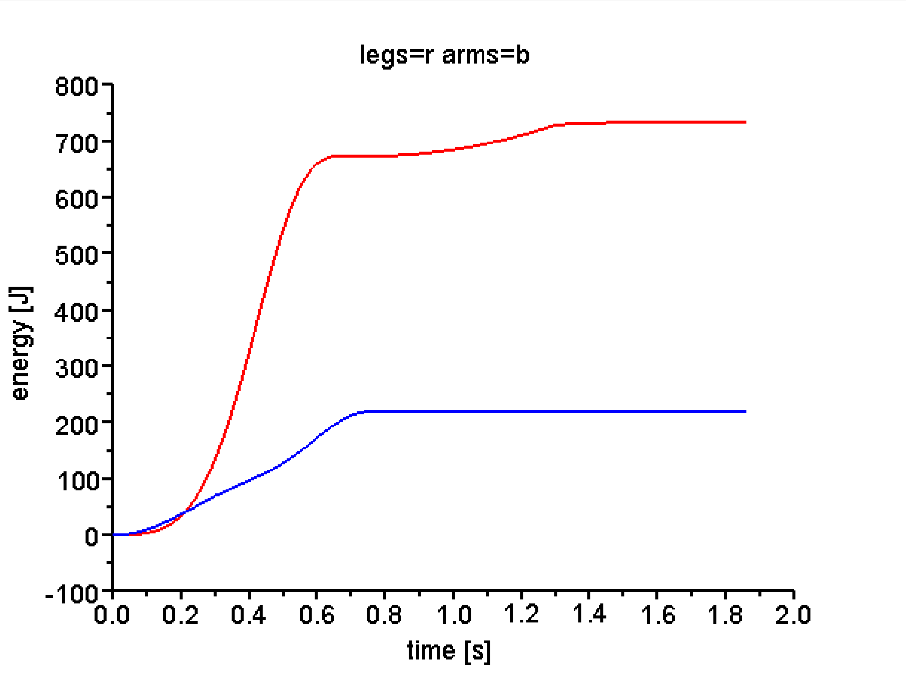

Fig 3.6
Energy delivered by arms and legs during one cycle.
red: legs
blue: arms

In Fig 3.6 the result of the integration of the power curves of arm and legs is shown.

Unfortunately, the colour codes are not the same as in Fig 3.5.

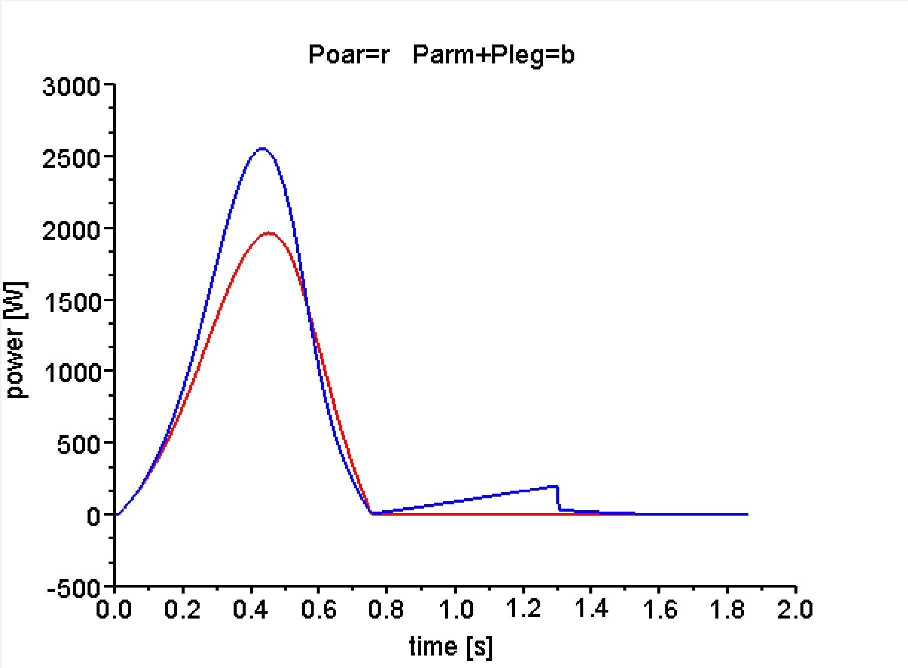

Fig 3.7
Power distribution.
blue line: power of arms + legs
red line: power to the oars

The above graphs all referred to the power (energy) sources arms and legs. The following graph shows the sinks of power, blade losses and hull friction, and the changes in kinetic energy in rower and hull.

The end points of the blue and the red sum up to the total energy consumption during one cycle. The zero level of kinetic energy is the situation at the start (and thus at the end) of the cycle. Therefore negative kinetic energies are possible.

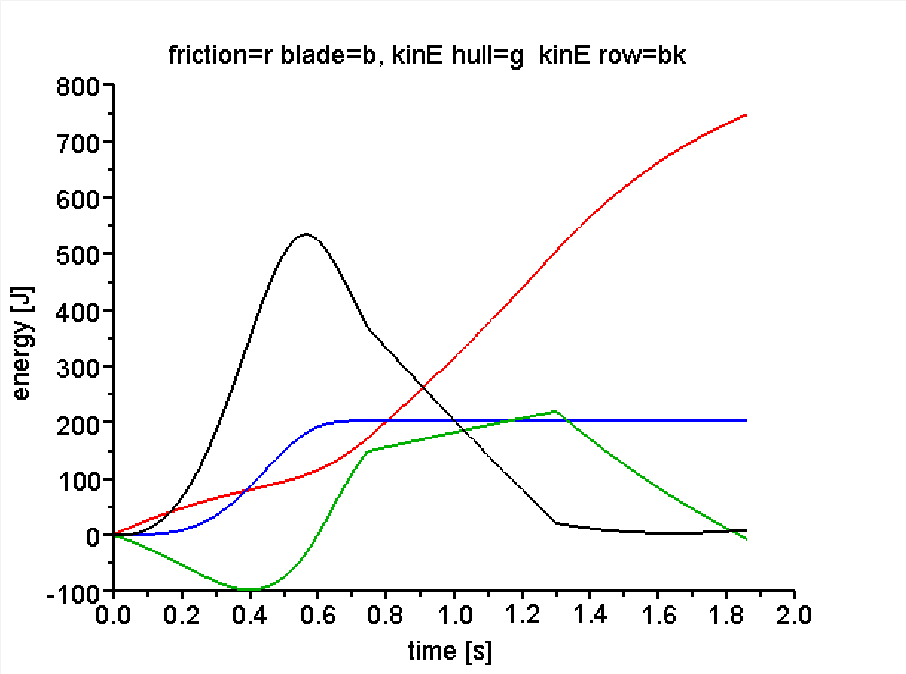

Fig 3.8
Power sinks.
red: hull friction
blue: blade losses
black: rower

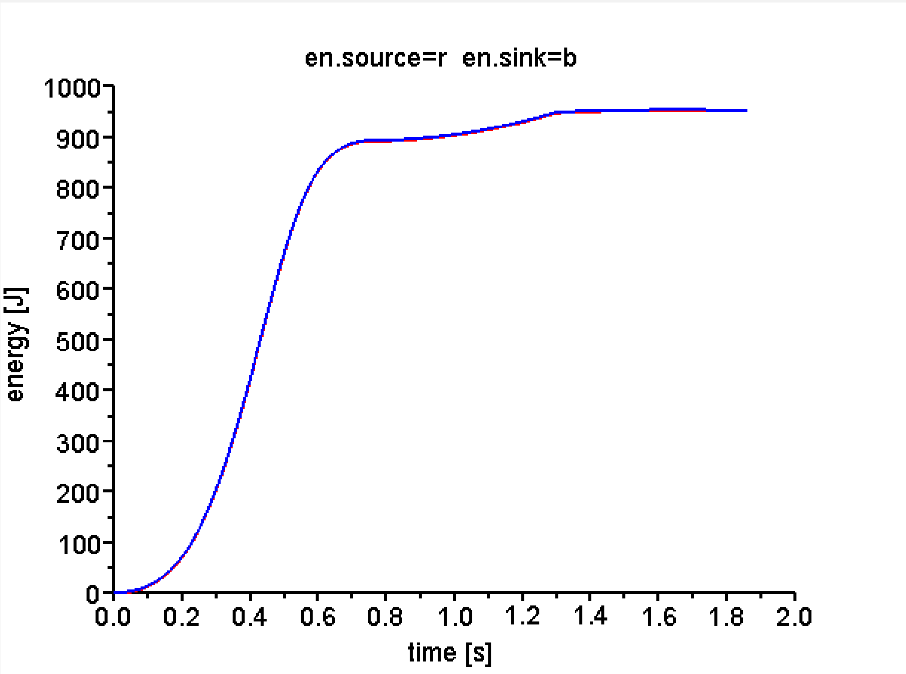

Fig 3.9
Energy sources and –sinks in one graph.
red: sources
blue: sinks

Fig 3.9 shows the integrated power curves for sources and sinks. They cover each other almost perfectly as should be the case.
Table 3.3 presents the energy balance in numbers. The left part of the table the unit is Joule, in the right part of the table the energy figures are divided by the total cycle time to produce the energy balance in terms of power, unit Watt.

SINKS	Joule	Watt		SOURCES	Joule	Watt
hull	747.868	402.079		arms	218.730	117.597
blade	203.089	109.188		legs	733.02	394.096
	950.958	511.267			951.750	511.693
kinetic	0.650	0.349				
	951.608	511.616				
rounded	951.7	511.6			951.7	511.6
internally dissipated	1.925	1.035		negative work	1.925	1.035
total	953.6	512.6			953.6	512.6

Table 3.3
Energy balance

The first rows of the table present the conservative case. Source and sink energies should be equal. This not completely the case 950.608 against 951.750, but it appears that the final kinetic energy is 0.650 J more than the start kinetic energy. This amount is added to the sink energy. The remaining difference is most probably due to calculation inaccuracy. Both source and sink energy are then rounded to 951.7 J. Internally dissipated energy or negative work is finally added on both sides of the balance.

4.Final remarks

The presented results give a reasonable insight into the energy bookkeeping during one cycle in the stationary situation.
The model of the rower as described in chapter 2 is not complete and perfect. A first refinement to be considered is a better description of the rower's back. The purpose of the exercise was however to be as realistic as possible with simple means.
In contradiction to what was found in Energy balance in a simple model no situation has occurred where the "engines" (power sources) absorb power instead of delivering it. There is one exception, see Fig 3.5. The blue line, power delivered by the legs, is for a very short time below zero at about 0.7s. It reaches only small negative values. The negative work is 1.925 J, see table 3.3. Also Atkinson pays much attention to this "reversed" situation but his conclusion is that negative work is of a substantial magnitude. The discussion on this subject has not yet come to an end. Top

5. Heavy shell

The question to be addressed in this section is: Will the internally dissipated energy considerably increase for a greater shell mass m1?

Considered is the model of a single wherry. The single wherry is endangered specie but was once popular in recreational rowing. It has a heavy wooden hull, one rowing seat and a comfortable cox seat. It was usually rowed by a man and coxed by a woman. Despite its recreational character it was sometimes raced.

Most of the system parameters used above were maintained, the following have been changed:

m1 = 130 kg (shell + cox)
c = 4.5 N.s2/m2 (hull resistance coefficient)
TREC = 1.5 s (time for the recovery)

See the energy balance and compare with Table 3.3.

SINKS	Joule	Watt		SOURCES	Joule	Watt
hull	621.05	267.12		arms	286.78	123.34
blade	217.37	93.49		legs	549.64	236.41
	838.42	360.61			836.42	359.75
kinetic	0.170	0.073				
	838.59	360.68				
rounded	838.5	360.7			838.5	360.7
internally dissipated	15.3	6.6		negative work	15.3	6.6
total	853.9	367.3			853.9	367.3

Table 5.1
Energy balance

The rounding of errors is handled in the same way as in Table 3.3.

The conclusion is: For the heavy shell the internally dissipated energy is considerably increased but compared to the total amount of energy delivered by the rower it remains small.