Theory and Model

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Preface
The first model of the author was presented in 1996 using Matlab/Simulink. From time to time it has been updated and revised. The history of these updates/revisions will not be discussed here. Unfortunately my Matlab licence has expired. The programs have been converted to Scilab. Visitors having access to Matlab/Simulink can still receive the source codes in Matlab when requested. Version conflicts are not unlikely to occur and I cannot give support anymore. Scilab code can be received instead.

Results of systematic variations of the model parameters are available by clicking here.
The author does not have the intention to convince the reader of the correctness of his model. He offers this material to the rowing community. He will be satisfied when it is read critically and it would be nice when a few people could appreciate it. (In the mean time this has happened).
The reader is free to copy and to use the material on this web page. Proper referencing will be appreciated. The author will gladly respond to questions and suggestions.

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1 Introduction
Rowers, scullers and oarsmen, like to talk about their favorite sport and produce the most remarkable ideas about rowing style and equipment. Maybe these discussions contribute to the attractiveness of rowing. Quantitative contributions in these discussions are rare. Although

Newtonian mechanics are sufficient to uncover all the secrets of the system boat/oars/rower feelings and visual observation of the relative motion of the components can easily lead to misunderstanding. On the web quite some material on rowing can be found. To mention just a few: Anu Dudhia: Physics of Rowing, (strongly recommended for developing an understanding of the basics of rowing), Atkinson: Rowing and the Rowing Biomechanics Newsletter.
Dudhia deals with a great number of items that he treats as, more or less, isolated problems. Atkinson has a complex model containing legs, arms and body of the rower. He shows a perfect understanding of the mechanics of rowing. The Biomechanics Newsletter is issued monthly by the author Valery Kleshnew of the English Institute of Sport and contains a wealth of field data. The mentioned authors present a great number of links to other web pages. I have learned a lot from these authors. The reason why I think that I can add something to the discussion is that a rather simple model that nevertheless contains the essential parts of the real live system can fruitfully be used for parametric studies. And because it gave me a lot of pleasure. In the course of the years a number of student-rowers have asked my advice for their thesis.
Some tuning of the presented model has been performed to obtain e.g. realistic values for speed, power input and the frequency of the rowing cycle. It is a very strongly simplified model of the real system but it fulfills the requirements as stated above. It is restricted to the steady state rowing cycle. It is a purely mechanical system with distinguishable parts. The model contains a coupling between the kinematics of rower and oars on the one hand and the forces on the blade on the other hand. You can consider the model as the model of a rowing machine that has much (or at least something) in common with a boat propelled by a human rower.
Anticipating on the results of the simulation the conclusion is that, as all insiders know already, no superior rowing style exists.
Rowing is and remains a matter of pulling as hard as you can.

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2 Hydrodynamic forces on the blade.
Lift- and drag forces are calculated as a function of relative fluid velocity and angle of attack.
The treatment of these forces is explained under Lift and Drag on this web site.

$$F_D = 0.5\, C_D\, \rho\, A\, u^2 \qquad F_L = 0.5 C_L\, \rho A u^2$$

F_D = drag force on the blade
F_L = lift force on the blade
u = fluid velocity with respect to the blade
C_D, C_L = drag and lift coefficient, dependent on the angle of attack
A = blade area, ρ = density of water
The force on the blade is worked out further in section 6.

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3 Definition of the model
In order to avoid referring to various types of boats the discussion will focus on the single scull. Apart from the selection of the parameters this is not a restriction in the generality of the model.
The model consists of two masses:

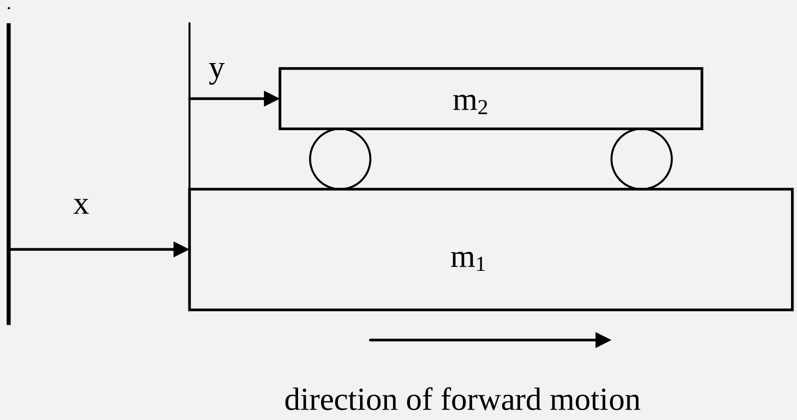

Fig 3.1
Masses and coordinates in the system

m_1= mass of the boat + that part of the mass of the rower that does not move with respect to the boat + hydrodynamic added mass
m_2= that part of the mass of the rower that moves with respect to the boat
x = absolute coordinate of the boat
y = relative coordinate of the mass m_2 with respect to the mass m_1
z = absolute coordinate of the joint centre of mass of m_1 and m_2
y = 0 when the mass m_1 is in the position of the catch
The relation between x, y and z is given by:

$$z(m_1 + m_2) = xm_1 + (x + y)m_2$$

$$z = \frac{m_1}{m_1 + m_2} x + \frac{m_2}{m_1 + m_2}(x + y)$$

$$z = x + \frac{m_2}{m_1 + m_2} y$$

and:

$$x = z - \frac{m_2}{m_1 + m_2} y$$

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4 Equation of motion

$$(m_1 + m_2)\ddot{z} = F$$

F = the sum of all external force components in the forward direction

$$F = F_1 + F_2$$

F_1 = the water resistance on the hull of the boat (including air resistance)

F_2 = component in the foreward direction of the force on the blade

$$F_1 = -C_1 \dot{x}^2$$

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5 Kinematics

See fig 5.1.

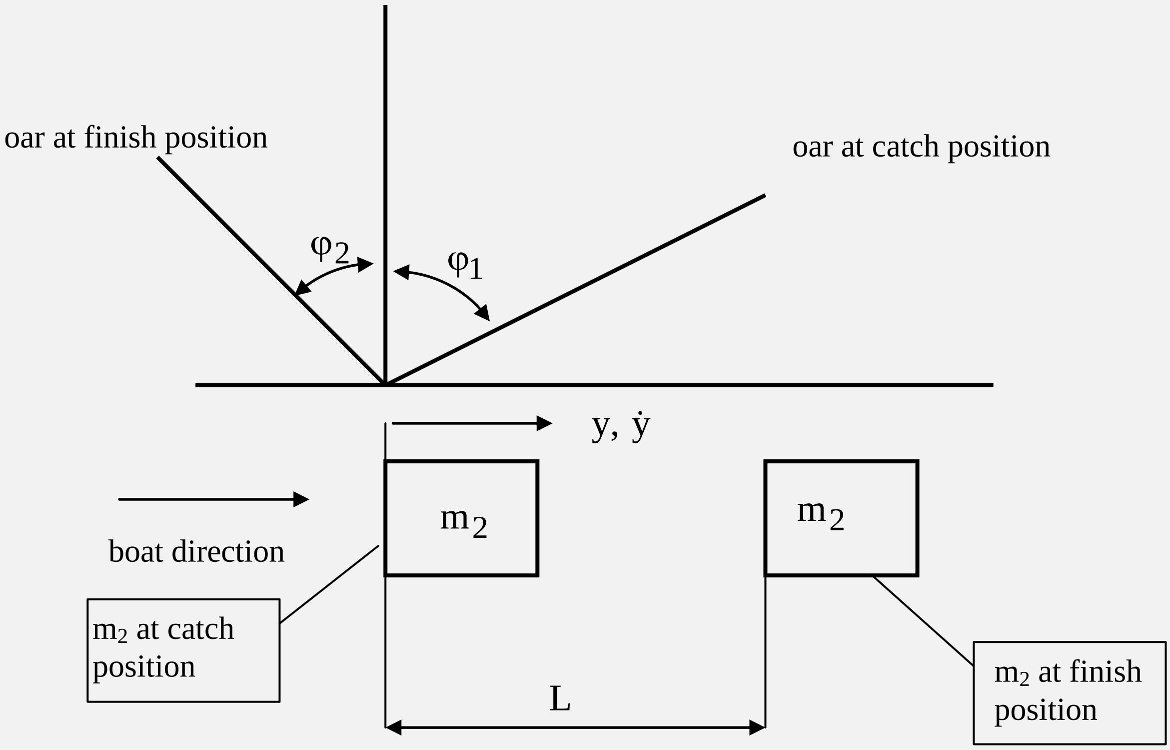

Fig 5.1
Catch and finish of oar and seat

The main geometrical parameter that describes the state of the model is the oar angle φ. The angle is measured with respect to the perpendicular through the pin. $\varphi_1 < 0$ is the catch angle and $\varphi_2 > 0$ is the finish angle. The total swept angle is $\varphi_t = \varphi_2 - \varphi_1$

The coordinate of the seat is y; $y = 0$ for $\varphi = \varphi_1$ and $y = L$ for $\varphi = \varphi_2$. The seat travels over a track of length L. The track L is taken "arbitrarily" somewhat longer than the actual sliding because the motion of head and shoulders is reduced to the seat motion. Seat motion, during the drive, starts and finishes with oar motion. An additional requirement is that the seat motion starts and finishes with zero speed. To describe the seat position the following function is used:

$$y = \frac{L}{2}.[1 - \cos(\frac{\pi}{\varphi_t}.(\varphi - \varphi_1))]$$

Find the seat velocity during the dive by differentiation:

$$\dot{y} = \frac{L}{2}.\frac{\pi}{\varphi_t}.\sin(\frac{\pi}{\varphi_t}.(\varphi - \varphi_1)).\dot{\varphi}$$

The relative position of mass m_2 with respect to the rotation point of the oar in Fig 5.1 does not have any meaning in the model.

The seat speed graph during the recovery is triangular (and negative). See Fig 5.2

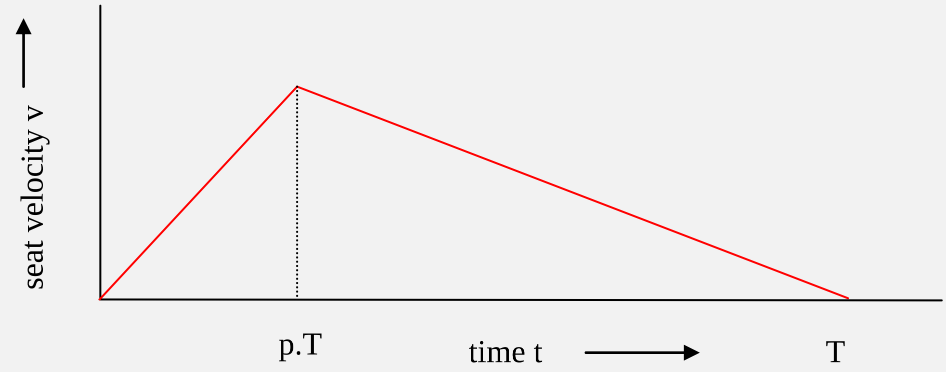

Fig 5.2
Seat velocity during the recovery as function of time

The time for the recovery is T. For t = 0, y = L and for t = T, y = 0. During the recovery the seat speed is negative. The maximum seat speed v_{max} ($v_{max}<0$) is reached after p.T ($0<p\leq1$) and $v_{max} = -2L/T$, the seat position is then $y = L(1-p)$.

The seat accelerations are:

$$a_1 = \frac{v_{max}}{p.T} \quad (a_1 < 0)$$

$$a_2 = -\frac{v_{max}}{T(1-p)} \quad (a_2 > 0)$$

The seat speed and seat position follows from:

$$t < p.T$$
$$\dot{y} = a_1.t$$
$$y = L + 0.5a_1.t^2$$

$$t > p.T$$
$$\dot{y} = v_{max} + a_2(t - p.T)$$
$$y = L(1 - p) - 0.5\, a_2\,(t - p.T)^2$$

The velocity of the blade is composed of two contributions: the velocity of the boat v_B and the velocity due to the rotation of the oar $\dot{\varphi}.L$. See Fig 5.4. (Fig 5.3 does not exist)

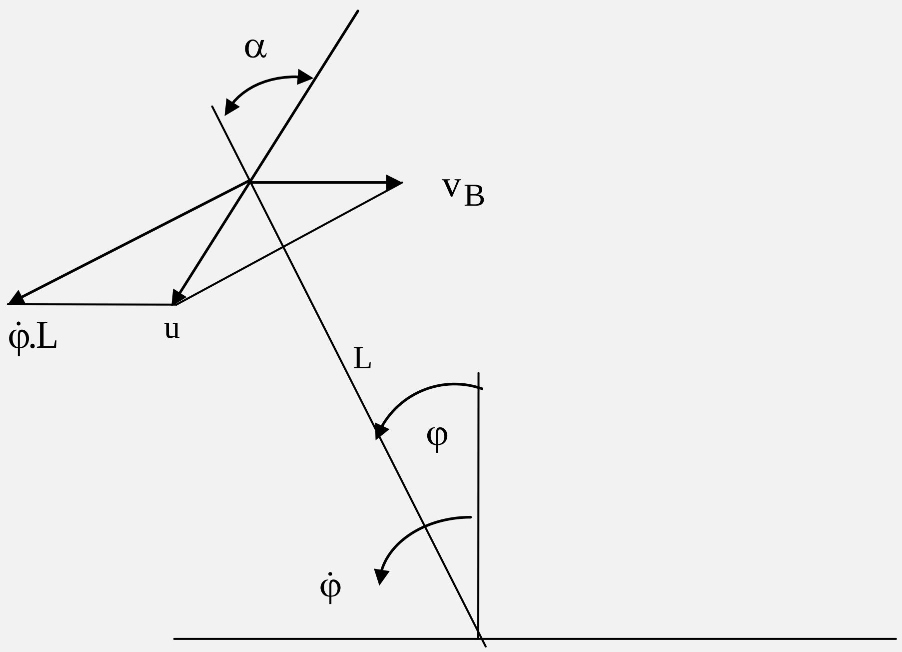

Fig 5.4
Velocities of the blade

They are decomposed in components perpendicular to the oar u_l, and parallel to the oar u_p. See Fig 5.5.

$$u_l = \dot{\varphi}.L - v_B.\cos\varphi \quad \text{and} \quad u_p = v_B.\sin\varphi$$

The resulting fluid velocity is then

$$u = \sqrt{u_l^2 + u_p^2}$$

The angle of attack follows from:

$$\alpha = \arctan\frac{u_l}{u_p}$$

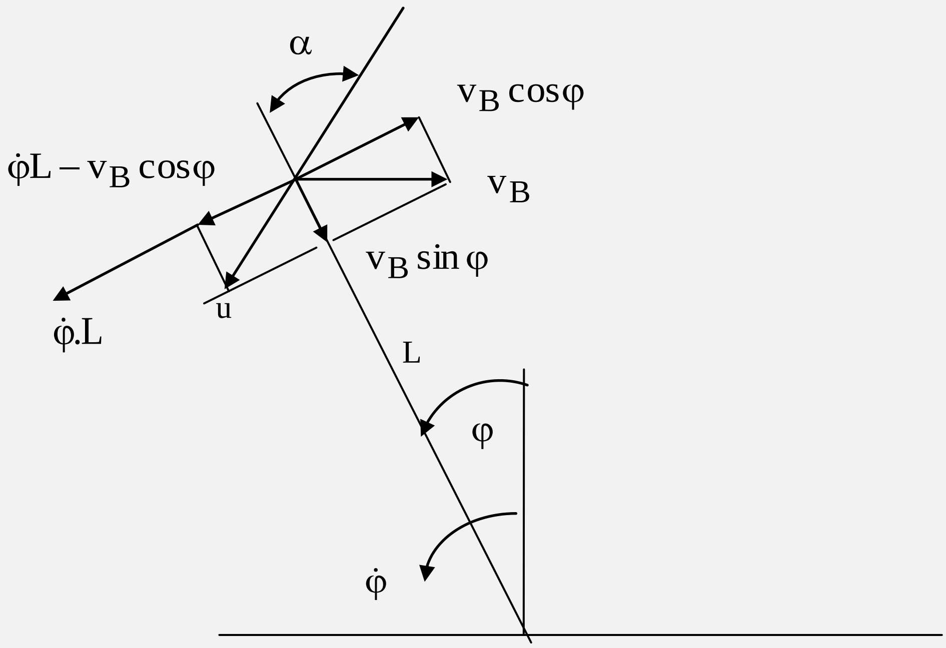

Fig 5.5
Velocities of the blade, decomposed

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6 Force on the blade

We had before:

$$F_D = 0.5\, C_D\, \rho\, A\, u^2 \qquad F_L = 0.5 C_L\, \rho A u^2$$

$$F_n = \sqrt{(F_D^2 + F_L^2)}$$

With F_n the resulting blade force perpendicular to the blade.
By iteration F_n is made equal to the required blade force. See Fig 6.1

The coefficients C_D and C_L are selected on the basis of α and are based on

Journal of Sports Sciences, April 2007; 25(6): 643-650
Nicholas Caplan & Trevor N. Gardner,
"A fluid dynamic investigation of the Big Blade and Macon oar blade designs in rowing propulsion"

The following expression for C_D and C_L are derived from the results in this paper but are the interpretation of this author:

$$C_D = 2\, C_{L\,max}\, (\sin\alpha)^2$$

$$C_L = C_{L\,max}\, \sin(2\alpha)$$

These expressions fulfil the requirement

$\frac{C_D}{C_L} = \tan\alpha$, which results in a blade force perpendicular to the blade

Caplan and Gardner found values $C_{Lmax} \approx 1.2$ for a flat plate *and* for a hatchet or big blade. This results in a maximum C_D value of 2.4. This is more than found in the literature. Therefore in our calculations we use maximum values of 2.0 and 1.0 for drag and lift respectively.

In the program, during the integration of the equation of motion, the force on the blade is found from the function describing the required blade force. By iteration the corresponding angular velocity of the oar is determined.

The unit required blade force is defined by four points as a function of φ in such a way that the blade force vanish for $\varphi = \varphi_1$ and $\varphi = \varphi_2$. The actual required force is obtained by multiplying the graph values by the maximum blade force F_{max}. This is the black line in Fig 6.1. This function is smoothed by spline interpolation, the red line.

Fig 6.1 shows a graph of the force for $-\varphi_1 < \varphi < \varphi_2$ with φ_1 = -1.1 rad, φ_2 = 0.6 rad.

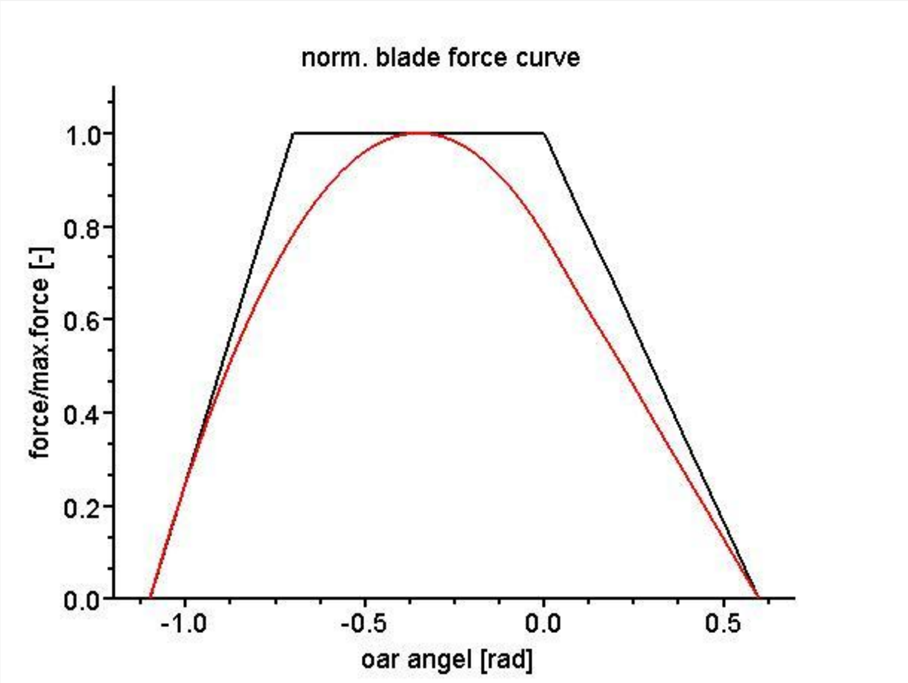

Fig 6.1
Normalized required blade force

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7 Obtaining a stationary situation

The equation of motion is integrated using an explicit integration process somewhat like the Newmark-ß method. The Newmark method deals with second order ODEs and here we are actually dealing with a first order system: only velocity and acceleration are present in the equations, the displacement is absent. Unconditional stability has not been proved but the behaviour of the integration process gives confidence. Starting in an arbitrary starting position integration in the time domain is carried out until a stationary situation has been obtained. Simulations have been carried out using a time step of 0.005 second during 10 seconds (model time). Integration is over the z-acceleration. z* is an estimate of the next value of the acceleration. See Fig 9.1.After the 10 seconds (and before) the stationary situation is obtained.
After completion of the integration the last complete cycle is selected for further processing.

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8 Energy

P_x is the power dissipated by the resistance of the hull in the water (and the air). The total amount of energy thus dissipated during the last cycle is E_x.

$$P_x = C_1 \dot{x}^3 \quad (\dot{x} \equiv v_B)$$

$$E_x = \int_0^T P_x dt$$

This energy is considered to be the useful energy.

where T = total cycle time (sorry, it was used for recovery time before)
The energy flow at the blade (work done by the force on the blade per unit of time) is P_{bl}.
The work done by the blade during the last cycle is E_{bl}.

$$P_{bl} = F_n (\dot{\varphi} L - \dot{x} \cos\varphi)$$

$$E_{bl} = \int_0^T P_{bl} dt$$

Where $P_{bl} = 0$ during the recovery.

This energy is considered as waste energy.

The average power consumed by the system is then

$$P = \frac{E_x + E_{bl}}{T}$$

(Possible internal energy dissipation has not been considered. A special chapter on the energy balance will be added).
The cycle efficiency (see above for the definition of useful and waste energy; cycle efficiency refers to the mean efficiency for one complete cycle) is defined as:

$$\eta = \frac{E_x}{E_x + E_{bl}}$$

The instantaneous propulsion efficiency is calculated from $\eta = \frac{P_u}{P_{in}}$

where:

$$P_{in} = F_n.\dot{\varphi}.L$$

$$P_u = F_n.v_B.\cos\varphi$$

P_{in} = power supplied to the oar
P_u = power used for the propulsion of the boat

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9 Programming

The main algorithm is performed by the sript *simmod.sce*. The main flow of *simmod* is displayed in Fig 9.1. *simmod* calls a function *main.sce.* It's arguments are the oar angular velocity (old value), oar angle, boat speed and seat position.
The output is: oar angular velocity (updated value), force on the blade and seat speed. Within the function a branch “drive” and a branch “recovery” exists.

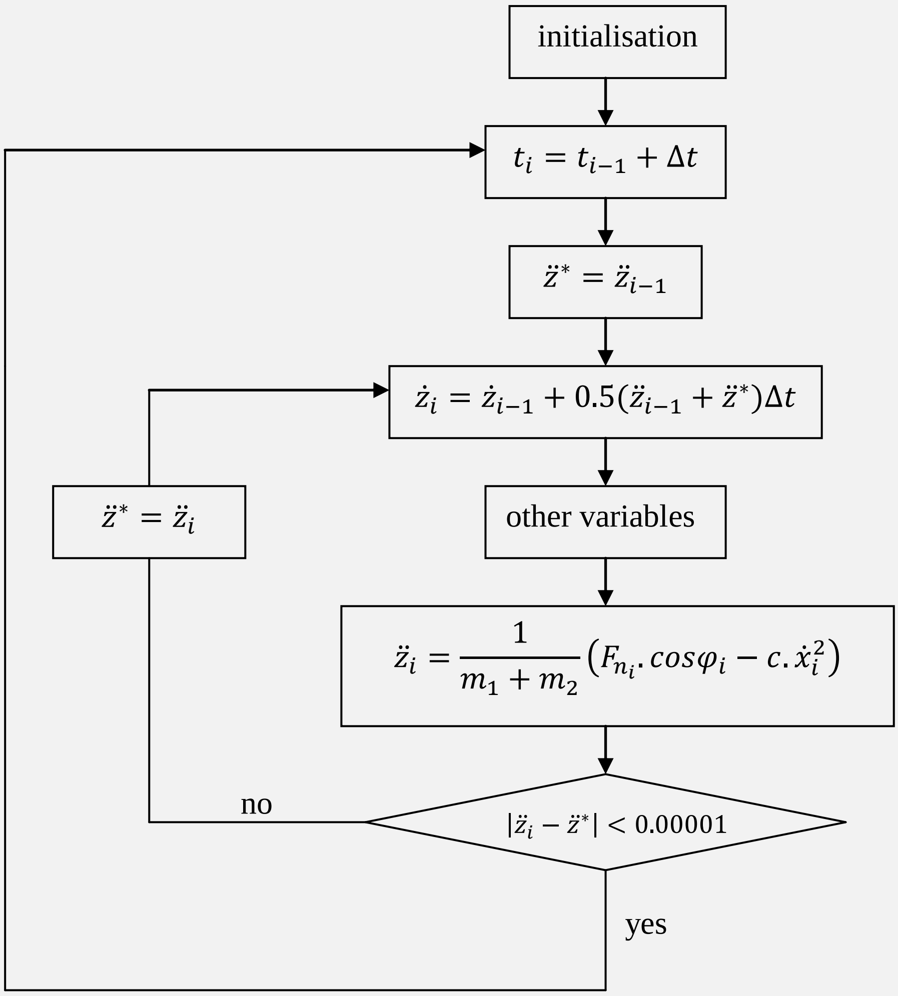

Fig 9.1
Script for Simmod.sce

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