Internal Work

home

1. Introduction

[....] the time has come to estimate the internal energy expended by a rower owing solely to his efforts on the slide; that is, the energy required to move himself to and fro without doing any external work [....]. This "internal" work seems to be the big, mysterious unknown in rowing calculations.

(I copied this introduction sentence from an e-mail received from Bill Atkinson)

In the following analysis this problem is addressed with a simple model that represents the situation in the recovery phase only. The drive phase is more complicated because kinetic energy in the system can be transferred to the oar handle to do work.

2. Model and theory

Fig 2.1 shows two masses as already used in other chapters. Fig 2.2 shows the forces on the masses using d'Alembert's principle.

m_1 = boat mass
m_2 = rower's mass
x = absolute coordinate of the boat
y = relative coordinate of the rower with respect to the boat (at the start of the recovery y = L, L = sliding distance)
z = absolute coordinate of the joint CoM of both boat and rower
T = cycle time, $\omega = 2\pi/T$
C = friction coefficient of hull
W = hull resistance
F_l = (leg) force between m1 and m2

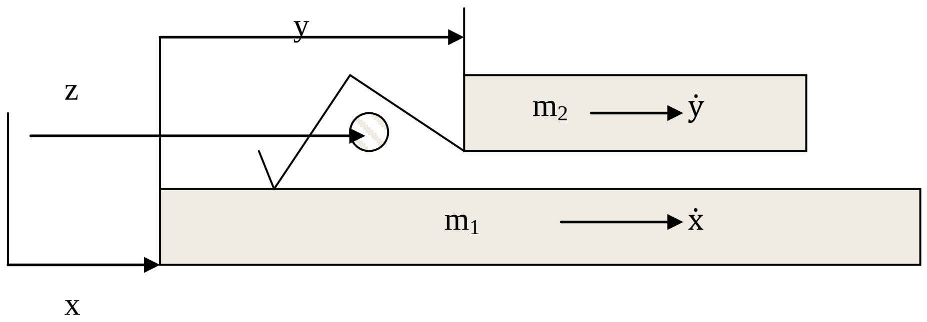

Fig 2.1

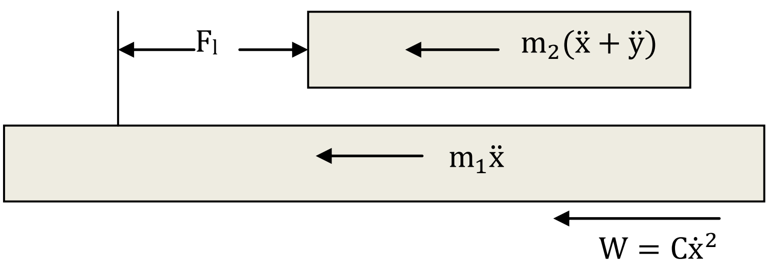

Fig 2.2

To define the z-coordinate we write:

$$z(m_1 + m_2) = xm_1 + (x + y)m_2$$

This expression also yields for the 1[rst] and 2[nd] derivative of x, y and z.

Solve for z-acceleration:

$$\ddot{z} = \frac{\ddot{x}m_1 + (\ddot{x} + \ddot{y})m_2}{m_1 + m_2}$$

but also:

$$\ddot{z} = \frac{W}{m_1 + m_2} = \frac{-C\dot{x}|\dot{x}|}{m_1 + m_2}$$

and:

$$\ddot{x} = \frac{(m_1 + m_2)\ddot{z} - m_2\ddot{y}}{m_1 + m_2} = \ddot{z} - \frac{m_2}{m_1 + m_2}\ddot{y}$$

the forcing function is the motion of m_2:

$$\ddot{y} = -0.5L\omega^2\cos\omega t$$

from which follows:

$$y = 0.5L(1 + \cos\omega t)$$

which yields: y = L for t=0, m_2 is at the end of the sliding when the recovery starts and

$$\dot{y} = -0.5L\omega\sin\omega t$$

The motion problem is solved by the Simulink system in Fig 3. The blocs with the shadow represent the solution of the equation of motion the other blocs are for checking and passing data to the Matlab command domain.
In the model x1 and x2 means $\dot{x}$ and $\ddot{x}$ and the same for y and z.
Function bloc y2 contains $\ddot{y} = -L\omega^2\cos\omega t$.
Function bloc x2 contains :

$$\ddot{x} = \ddot{z} - \frac{m_2}{m_1 + m_2}$$

The blocs Tw... bring the variable mentioned *in* the bloc to the Matlab work space.

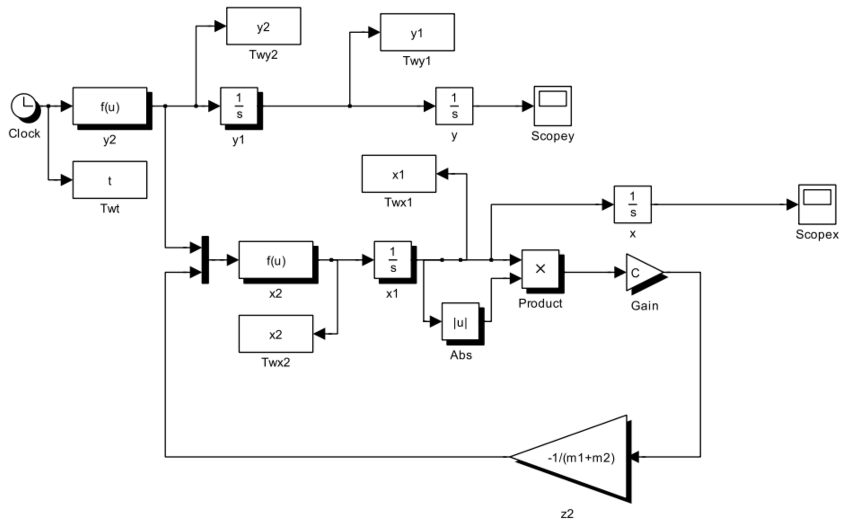

Fig 2.3
Zoom in for reading the text in the model.

In Matlab are performed the following calculations:

Leg force: $F_l = m_2.(\ddot{x} + \ddot{y})$
Leg power = leg force times extension speed of the legs: $P_l = F_l.\dot{y}$
Power dissipation due to hull friction: $P_h = C\dot{x}^2|\dot{x}|$
Energy delivered by the legs during the recovery: $E_l = \int_0^{T/2} P_l dt$
Energy dissipated by hull friction: $E_h = \int_0^{T/2} P_h dt$
Power and energy of the legs has been split in positive (i.e. delivered by the legs) and negative (i.e. absorbed by the system when the system is considered to be conservative) parts.

The kinetic energy in the system is the sum of the kinetic energy of m_1 and of m_2.

$$E_{k1} = 0.5m_1\dot{x}^2 \text{ and } E_{k2} = 0.5m_2\,(\dot{x} + \dot{y})^2$$

$$E_k = E_{k1} + E_{k2}$$

The rate of change of the kinetic energy, increase or decrease of energy is a power

$$P_{k1} = \frac{d}{dt}E_{k1} = \frac{d}{dt}(0.5m_1\dot{x}^2) = m_1\dot{x}\ddot{x}$$

$$P_{k2} = \frac{d}{dt}E_{k2} = \frac{d}{dt}(0.5m_2(\dot{x} + \dot{y})^2) = m_2(\dot{x} + \dot{y})(\ddot{x} + \ddot{y})$$

The power balance is $\Delta P = P_l - P_h - P_{k1} - P_{k2}$ must $\rightarrow 0$.

Calculations are performed for the following values of the parameters:

Kept constant	Varying	Unit
m1 = 30		kg
m2 = 70		kg
L = 0.7		m
T = 2.0		sec
	C = 0, 3.5	$N.s^2.m^{-2}$
	ICx1 = 0.0, 5.0	m/s
ICx1 = initial condition of boat velocity		

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3 calculations

3.1 The first case to consider is

initial speed = 0

hull friction = 0

T = 2 s

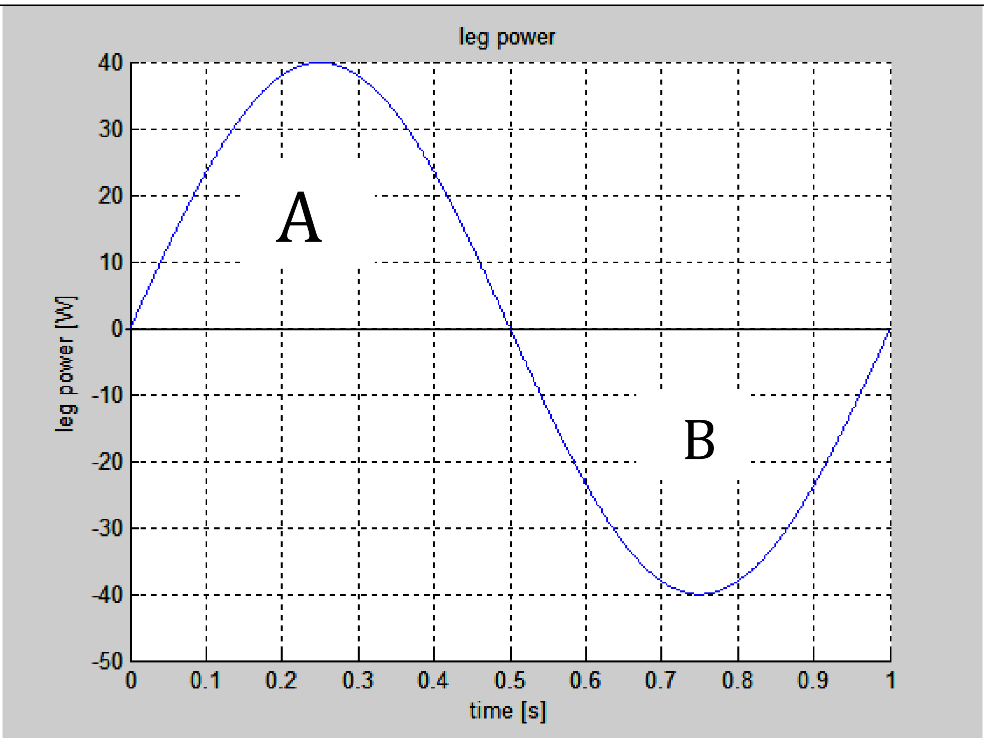

Fig 3.1 Leg power.
The area A represents the work done by the rower in the first phase of the motion. The area B represents the energy, *in a conservative system*, that the system feeds back to the rower. Area A = area B. In a nonconservative system area B represents the energy that is dissipated.

Discussion:

In a conservative system no work is done by the rower. The energy delivered in the first part of the recovery is recovered in the second part of the recovery. It is unlikely that this occurs in the legs of the rower, however, the model calculations do not decide on this particular question.

Let us call the energy represented by the area A, E_A ($E_A>0$) and the energy of area B, E_B ($E_B<0$), and the total energy delivered by the rower is E_l.

So at least three cases are to be considered:

$E_l = E_A + E_B = 0$ in this case

$E_l = E_A$

$E_l = E_A + |E_B| = 2*E_A$ in this case

$E_A = |E_B| = 12.7$ J. The energy delivered by the rower is then based on the above mentioned possibilities 0, 12.7, or 25.4 J.
In the following cases will be chosen for the second option. Only energy positively delivered by the rower will be counted, based on the following considerations:

According to this author it is most likely that E_B is dissipated in the muscles and articulations of the rower and disappears as heat out of the system. The first option requires elastic properties of human body. To a certain extend they will be present but it is difficult to assume enough capacity to store and pay back all E_B. In the third case energy will be consumed to compensate for other energy.

To be clear it is stated explicitly that in this case no external work has been done indeed. All the work done by the rower 0, E_A or $2*E_A$ is lost: at the end no kinetic energy is present in the system, the joint CoM has not moved, no work on hull friction has been done.

E_A has the same value as the expression in <u>click here</u>

$$E = 0.5\, \dot{y}_{max}^2 \frac{m_1 . m_2}{m_1 + m_2} = 12.7J$$

with

$$\dot{y}_{max} = 0.5L\frac{2\pi}{T}$$

A check of the power balance is carried out by plotting ΔP for each integration step. See Fig 3.2.

When and where is energy dissipated? Not in the first part of the motion. The legs do work, area E_A, and that work is converted into kinetic energy. No dissipation of energy. Clearly, the energy is dissipated in the second part of the motion. The legs act as a damper, kinetic energy is dissipated in heat. Suppose now (right or wrong) that this dissipation of energy does not require energy supply from the rower.
The energy loss is represented by area B. In this particular case it is immaterial whether we take magnitude of area A or area B because they are equal. But in next cases to be considered they are not. Let us agree upon that area B represents the energy loss.

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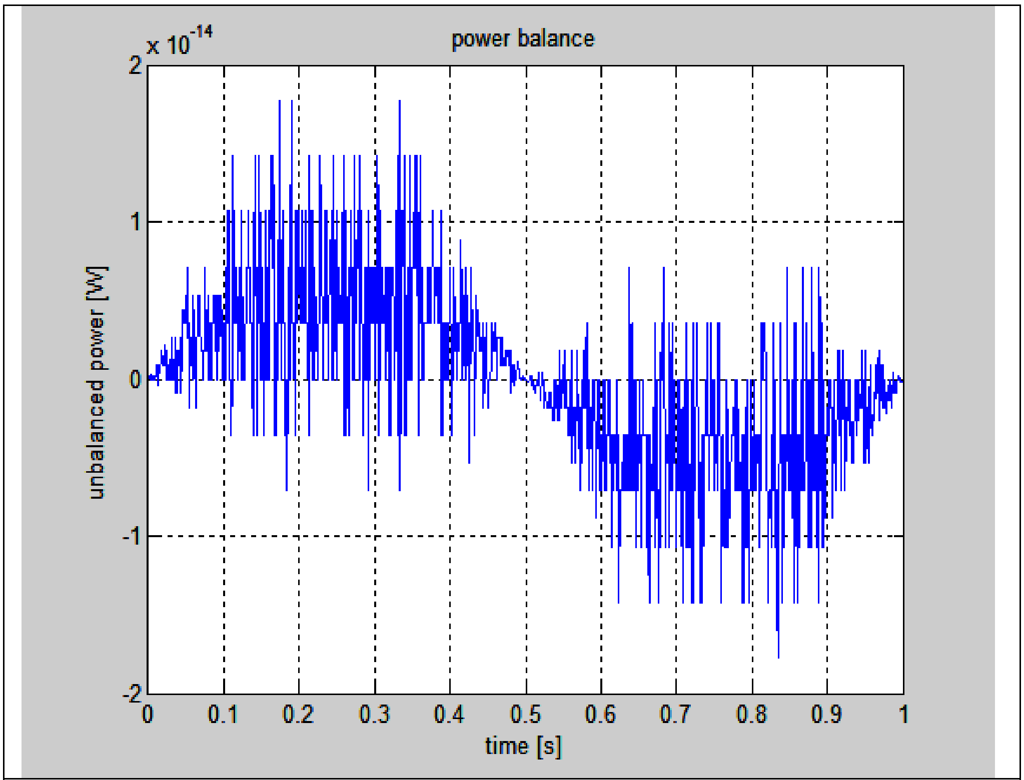

Fig 3.2 Power balance.
This figure shows that $\Delta P \approx 0$ for every time step (0.001s)

Finally Fig 3.3 presents the kinematic energy in the system.

Comparing the work done and the maximum kinetic energy in the system we find $0.5m_1\dot{x}^2 + 0.5(m_1 + m_2)(\dot{x} + \dot{y})^2 = \int_0^{0.5} P_l dt$ as expected. ($\dot{x}$ and $\dot{y}$ taken at $t = 0.5$ s)

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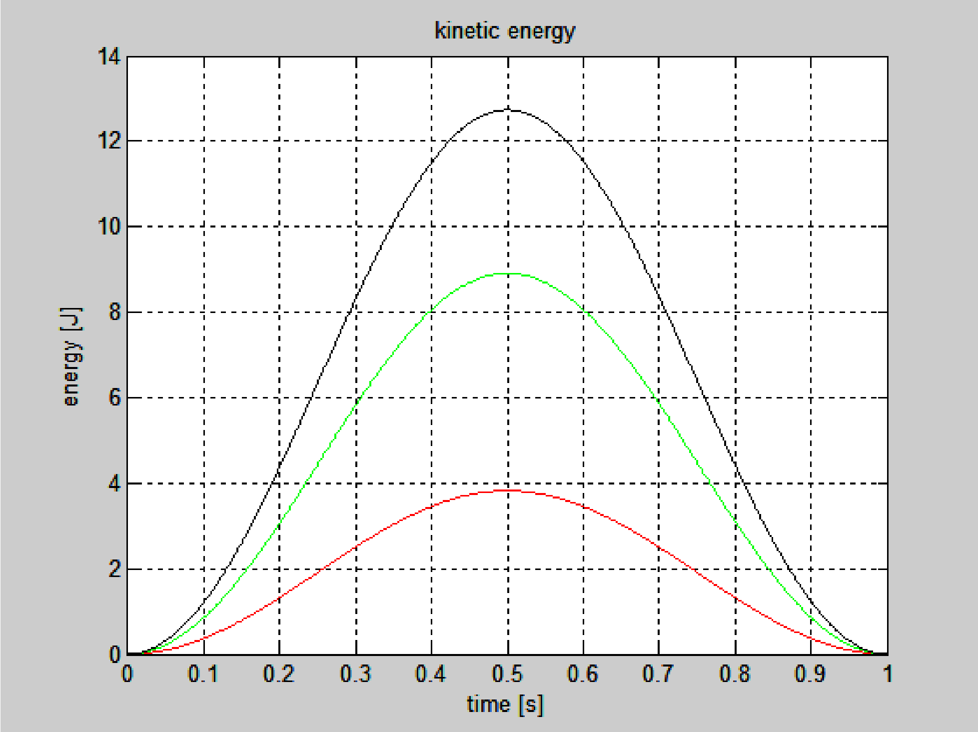

Fig 3.3 Kinetic energy in the system.
red: in m_2, rower
green: in m_1, hull
black: in $m_1 + m_2$, system

3.2 The second case to consider is:
initial speed = 0
hull friction coefficient C= 3.5 $N.s^2.m^{-2}$
T = 2 s
It appears that this case does not differ noticeable from the previous case. The reason is that the hull friction is very small because the hull velocity is small, maximum value = 0.76 m/s.
E_A = 13.03 J and E_B = 12.43 J. Energy dissipated by hull friction E_h= 0.6 J.
Note: E_A has increased and E_B has decreased with respect to case 1.

3.3 The third case to consider is:
initial speed = 5 m/s
hull friction coefficient C= 0
T = 2 s
This is not e very practical case but for the understanding of the system it is useful.
$E_A = |E_B|$ = 12.7 J exactly the same as for the first case with zero initial speed. Final speed = initial speed, hull friction loss is of course zero.
The difference between maximum kinetic energy (at t =0.5) and minimum kinetic energy (at t = 0) equals again the work done by the legs E_A. We conclude again that this

work is lost. At the end of the recover the velocity is the same as at the start and no external work has been done.

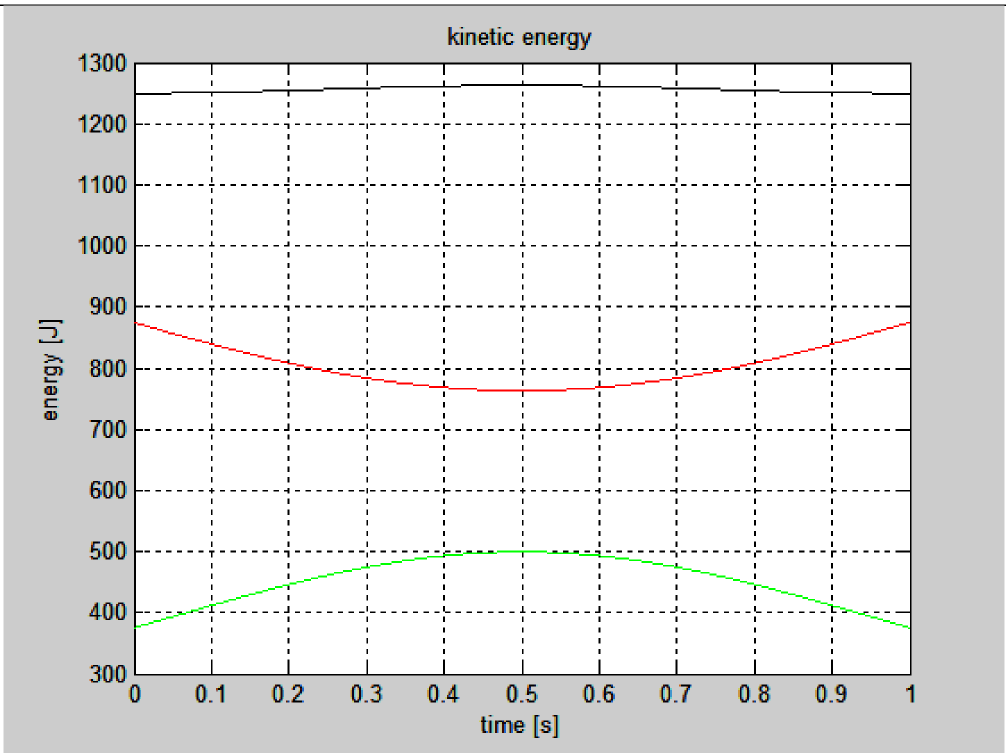

Fig 3.4 Kinetic energy in the system.
red: in m_2, rower
green: in m_1, hull
black: in $m_1 + m_2$, system

3.4 The fourth case to consider is:
initial speed = 5 m/s
hull friction coefficient C= 3.5 $N.m^2.s^{-2}$
T = 2 s

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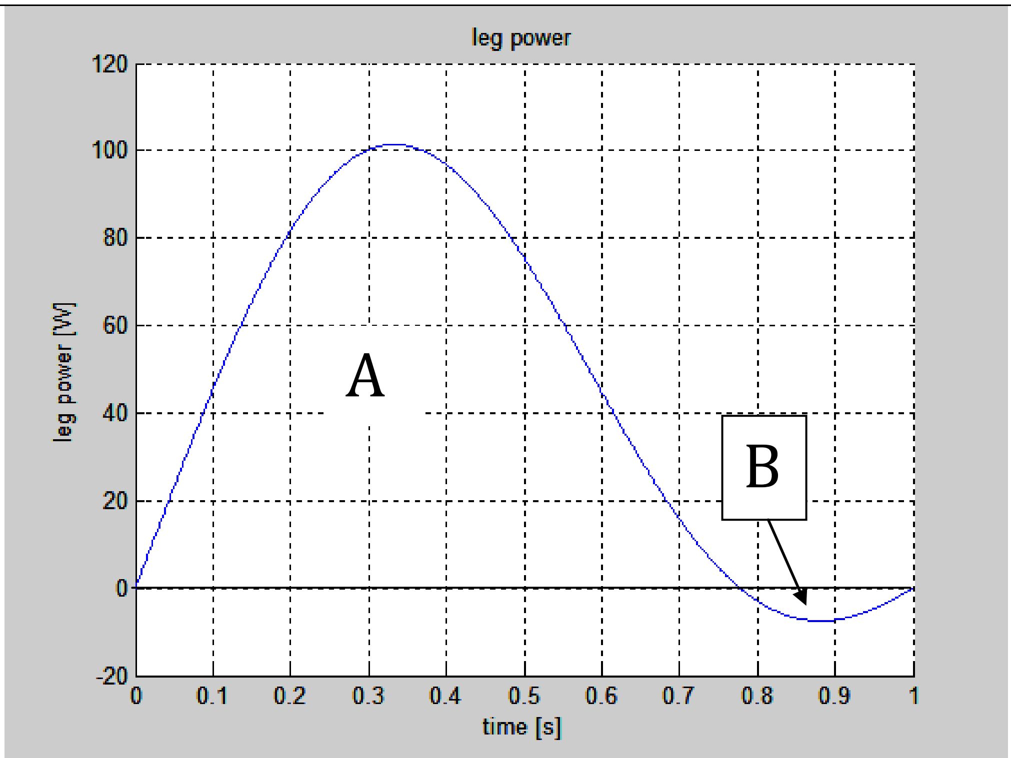

Fig. 3.5 Leg power.

Compare with Fig 3.5 and notice the difference in E_A =46.35 J and E_B = 1.97 J.

Summary energy

Positive work of rower E_A = 46.35 J

Negative work of rower E_B = 1.97 J

Energy by hull friction E_h = 449.38 J

Decrease of kinetic energy ΔE_k = 404.18 J

Energy balance $\Delta E = E_A - E_B + \Delta E_k - E_h = -0.82$ J $\rightarrow 0$.

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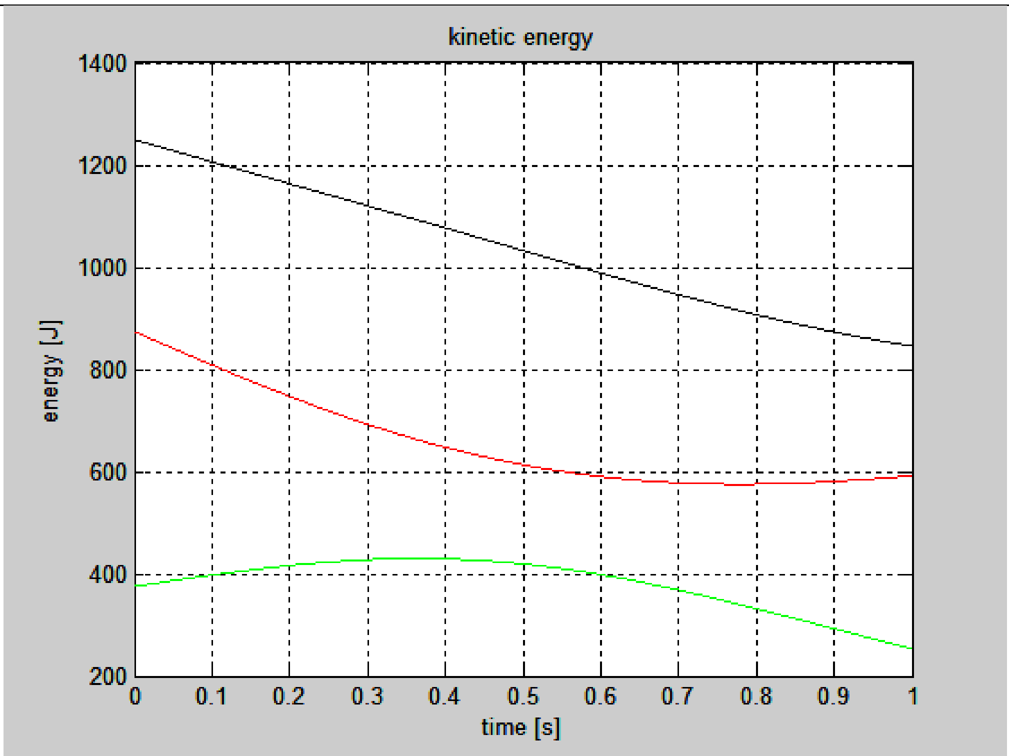

Fig 3.6 Kinetic energy in the system.

red: in m_2, rower
green: in m_1, hull
black: in $m_1 + m_2$, system
decrease of kinetic energy $\Delta E_k = E_k(t=0) - E_k(t=1) = 404.18$ J

4 Conclusion

Based on the presented model and calculations the conclusion would be that in frictionless system the rower delivers work that is dissipated internally. In a real system with external friction the amount of energy dissipated internally is very small and negligible for practical purposes.

For the stationary erg, Concept2, the situation is different. Click here