Lights Out Revisited
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Lights Out is a puzzle where all light bulbs in an array of 5×5 light bulbs must be switched off. The main article was published in CFF 68. This document provides some additional exercises.

Exercises for n=5

Exercise 1.
With a symmetry argument, you can find out for yourself what the changing positions are for rows 4 and 5. (Note that one can mirror the board in the middle column)

Exercise 2.
Show the correctness of the effects for row 5 of combined positions in row 1. They are mentioned here to give you an idea of the general process for higher values of n.

Exercise 3.
Since 2,3 in row 1 has the same effect as 4, one can conclude that not all positions for this order are solvable. (This is hard)

Exercise 4.
Show that when you start with only one light on at position 3 on row 1, that you can not put all lights off.

Position in row 1	Positions in row 5	
--------	---------	
1	2,3,5	
2	1,2,3	
3	1,2,4,5	
4	3,4,5	
5	1,3,4	
2,3,4	-	(2=3,4 3=2,4 4=2,3)
1,3,5	-	(1=3,5 3=1,5 5=1,3)
1,2	1,5	
1,4	2,4	(2,5=1,4)

With the above, we have the following equalities (that is that clicking the different sets of bulbs in row 1 has the same effect). Note that these may be used to simplify any partial solutions to a minimum number of bulbs to be clicked in row 1.

1,2,3 = 1,4
1,2,4 = 5
1,2,5 = 4
1,3,4 = 1,2
1,2,3,4 = 1
1,2,3,5 = 3,4

1,2,4,5 = -
1,2,3,4,5 = 3

Exercises for n=6
The marked positions are the ones that change only 1 bulb in row 6

Exercise 4
Show that when bulbs 5,2,3,4 in row 1 are pushed, then the whole row 6 is changed

Exercise 5
Show that you can put all lights off in any starting position in the case n=6.

Position in row 1	Positions in row 6
1	1,5
2	2,4,6
3	5*
4	2*
5	1,3,5
6	6,2
1,3	1*
1,5	3*
2,6	4*
4,6	6*
3,4,5	1,2,3
2,5	1,2,3,4,5,6

Exercises for n=7
The marked positions are the ones that change only 1 bulb in row 7

Exercise 6
Show that the table is correct.

Exercise 7
Show that you can put all lights off in any starting position in the case n=8.

Position in row 1	Positions in row 7
1	1,3,4,6,7
2	3,4,6,7
3	1,2
4	1,2,4,6,7
5	6,7
6	1,2,4,5
7	1,2,4,5,7

```
1,2          1*
1,2,3        2*
2,3,4        3*
3,4,5        4*
4,5,6        5*
5,6,7        6*
6,7          7*


2,4          1,2,3
2,3,4,5,6    1,2,3,4,5,6,7,8
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Exercises for n=8
The marked positions are the ones that change only 1 bulb in row 8

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Position     Positions
in row 1     in row 7
-------- ---------
1            5*
2            4,6
3            3,5,7
4            2,4,6,8

5            1,3,5,7
6            2,4,6
7            3,5
8            4*

3,5          1*
2,6          2*
1,7          3*
2,8          6*
3,7          7*
4,6          8*

1,2,3,5,6,7,8 1,2,3,4
5,7,4        1,2,3,4,5,6,7,8
```

Exercises for n=9
The marked positions are the ones that change only 1 bulb in row 9

Exercise 8.
A solvable position is either directly found or else by clicking bulb 1 in row one and working your way down again.

Exercise 9
What is the probability that a solution is found at once? Is it 1/2 or 5/9?

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Position     Positions
in row 1     in row 7
-------- ---------
1            1,3,5,7,9
2            - (nothing)
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3	1,3,5,7,9
4	-
5	1,3,5,7,9
6	-
7	1,3,5,7,9
8	-
9	1,3,5,7,9

Exercises for n=10
The marked positions are the ones that change only 1 bulb in row 10

Exercise 10.

Position in row 1	Positions in row 7
1	1,3,7,9
2	4,6,10
3	1*
4	2,4,6,10
5	7,9
6	2,4
7	1,5,7,9
8	10*
9	1,5,7
10	2,4,8,10
2,4	2*
1,3,4	3*
2,4,6	4*
3,5,7	5*
4,6,8	6*
5,7,9	7*
7,8,10	8*
7,9	9*
1,3,4,5,6,7	1,2,3,4,5
1,3,8,10	1,2,3,4,5,6,7,8,9,10

Exercises for n=11
The marked positions are the ones that change only 1 bulb in row 10

Position in row 1	Positions in row 7	Reducible to
1	4,5,6,10,11	1,2,4,5,6
2	3,4,6,7,9,10,11	1,2,4,5,6
3	2,3,5,7,9,10	2,3
4	1,2,4,6,7	1,2,4,5,6
5	1,3,5,6,8	1,2,4,5,6

6	1,2,4,5,7,8,10,11	-
7	4,6,7,9,11	
8	5,6,8,10,11	
9	2,3,5,7,9,10	
10	1,2,3,5,6,8,9	
11	1,2,6,7,8	

1,6,11	-
2,6,10	-
3,9	-
4,6,8	-

Complete set:

		symmetric	
3,4,5,6,7	1,2,4,5,6		
1,4,5,6	1,3,4,5,6		
2,4,5,6,7	1,11	y	
1,2,3	2,10	y	
2,5,6	2,9		
1,3,7	2,3		
1,2,3,5,6,7	3,9	y	
2,7	3,10		
1,4,6	4,8	y	
3,5,6,7	5,7		y
1,2,3,5,7	1,2,3,4,5,7,8,9,10,11		

Exercises for n=12

1	1,5
2	2,4,6
3	5,7
4	2,6,8
5	1,3,7,9
6	2,4,8,10
7	3,5,9,11
8	4,6,10,12
9	5,7,11
10	6,8
11	7,9,11
12	8,12

5,7,9	1
4,10	2
3,7,11	3
2,6,8,12	4
1,5,7,9	5
4,6,8,10,12	6
1,3,5,7,9	7
4,6,8,12	8
1,5,7,11	9
2,6,10	10
3,9	11
4,6,8	12